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were balloted for and duly elected Associates of the Society.

On the General Solution of the Problem of Disturbed Elliptic Motion. By E. Neison, Esq.

There are two distinct methods of dealing with the problem of disturbed elliptic motion. One method is to throw the effect of the perturbations directly on the three coordinates, or to suppose that the disturbed body moves in an irregular curve which is

approximately of the form of an ellipse. The other method is to throw the perturbations directly on the elements of the elliptic orbit, by assuming the body to move in an ellipse which is perpetually changing in form and position. The first method may be called that of perturbed coordinates, and has the advantage of requiring only three variables, but in turn the disadvantage of employing more complex differential equations. The second method, which may be termed that of perturbed elements, has the disadvantage of requiring six or seven variables, but requires a very much simpler form of differential equations.

What is required for astronomical purposes is the polar coordinates of the disturbed body. By making these polar coordinates the three variables of the first method, their values at any instant is at once obtained. But by the second method what is obtained is only the value of the elements of the ellipse in which the disturbed body is momentarily moving. These must be transformed into an expression for the required polar coordinates, and this necessitates extra work. Consequently there is this additional disadvantage in the method of perturbed elements.

Despite the disadvantages which have been pointed out, the method of perturbed elements is the simplest and most expeditious way of determining the perturbations of the order of the first power of the disturbing forces. For this reason it is admirably adapted for employment in the planetary theories, where powers of the disturbing forces are rarely required. In the lunar theory, however, it is absolutely essential to consider the square of the disturbing forces. But this renders most complex the known methods of determining the perturbations of the elements and necessitates an enormous increase in the labour. For instead of there being only six powers and products of the three variable coordinates, the second method involves the formation of no less than twenty-eight powers and products of the seven variable elements. The calculation of these powers and products is laborious and difficult. Therefore, under these conditions, the second method involves many times as much work as the first method.

For these reasons the earlier workers on the lunar theory —Damoiseau, Plana, Lubbock, Pontécoulant—employed the method of perturbed coordinates. The polar coordinates were taken for variables, and the differential equations were simplified by omitting all quantities of a higher order than it was intended to retain in the approximation, whilst by special artifices the amount of work was lessened. Poisson strongly advocated the method of perturbed elements, and, at his instigation, Pontécoulant employed it in certain parts of his lunar theory, but pointed out that to calculate the lunar inequalities by this method would be a work of immense labour. Hansen made use of a method which is really a modification of the method of perturbed coordinates, though he did not throw the perturbations directly

on the coordinates. In other respects his system of using his method was the same as that employed by his predecessors. In all these cases, therefore, special means were taken to obtain a solution which should be complete to a given order, but which could not be extended to any further order, and depended entirely on the special features of the problem. Lately, Hill, by making use of rectangular coordinates and a modified form of the differential equations, has successfully determined the value of the perturbations which are independent of the eccentricities. His method possesses some generality of form, and is not limited to the particular problem. It consists of an expression connecting together the different indeterminate coefficients. It is, however, an expression which gives a very complex form to the complete coefficient of the nth term, and is restricted to the coefficient of a small class of easy terms. Moreover, it yields the value of the rectangular coordinates and requires extra work to transform it into an expression for the polar coordinates. It is, however, to be remarked that for the definite purpose which its author had in view these are matters of indifference.

Delaunay, alone, has successfully made use of a method of throwing the perturbations on the elements of the elliptic orbit. He does not make use of the ordinary method of the variations of elliptic elements—that is, the method which has been employed by Pontécoulant and Le Verrier in the planetary theories, or which was advocated by Poisson for use in the lunar theory. Delaunay devised a most ingenious method, which really consists in determining the value of the elements which will make the disturbing function assume a given form ; a form which enables him to directly integrate the resulting differential equations. It is a most elegant and ingenious method, but is far more complex than any other, and involves an enormous amount of work. It is no exaggeration to say that it involves thrice the amount of work which would have been necessary to obtain the same result by the use of Pontécoulant's method. It is, moreover, purely a special method, a method depending on a long series of particular transformations, and it would be impracticable to obtain by its means a general expression for the perturbations of the coordinates or elements. On the other hand, Delaunay's method has an important advantage over any method involving the use of indeterminate coefficients, as Hill's or my own ; it is possible to separately determine the value of each single term in the perturbations without its being necessary to determine any other.

It has long been my opinion that, by choosing suitable variables, and by transforming the differential equations in a proper manner, it would be possible to obtain a complete general solution of the problem of disturbed elliptic motion ; that is to say, a solution which would be so far general that it would be possible to write down the perturbations to the nth order of the disturbing forces, and be independent of any particular values of

the elements. Such a solution would be obtained if it were possible to form a simple expression for the complete perturbations of the order of the nth power of the disturbing forces. It seemed to me that if such an expression could be formed, it would be an important contribution to the theory of disturbed elliptic motion, and in particular to the lunar theory. For several years, therefore, it has been a problem which has engaged much of my attention.

It was not long before it was seen that though it would be possible to obtain an expression for the perturbations of the order of the nth power of the disturbing forces, yet it was evident that it would be very difficult to prevent this expression becoming very lengthy and complex. This difficulty has now been overcome, and a very simple expression has been found for the perturbations of the order of the nth power of the disturbing forces.

In the present paper I propose to give a short account of the principal results, sufficiently full to enable them to be understood; but the details and minor developments are deferred to a subsequent communication, when I propose to apply the results to some important points in the lunar theory.

Let αR be the disturbing function when the elements of the orbit have their instantaneous values x, y, z, X, Y, Z, the coefficient α being supposed to be a constant, and let the general term of R be denoted by

$$\mathrm{R}\cos \mathrm{w} = (\Lambda)\, \mathrm{x}^{\mathrm{F}} \mathrm{y}^{\mathrm{G}} \mathrm{z}^{\mathrm{H}} \cos\{(\lambda) + f\mathrm{X} + g\mathrm{Y} + h\mathrm{Z}\}, \qquad (1)$$

where (Λ) and (λ) are supposed to be independent of the elements of the orbit, and to be absolutely constant.

Suppose R to denote the value of R when the elements have their constant mean values. Then R will be the same function of the mean elements, x, y, z, X, Y, Z, as R is of the instantaneous elements, x, y, z, X, Y, Z, so that the general term of R can be written in the form

$$R\cos w = (\Lambda)\, x^{\mathrm{F}} y^{\mathrm{G}} z^{\mathrm{H}} \cos\{(\lambda) + fX + gY + hZ\}. \qquad (2)$$

Let each element of the orbit be considered as the sum of its constant mean value, and either of a single periodical function which represents its perturbations, or of n periodical functions which collectively represent its perturbations, so that, for instance,

$$\mathrm{x} = x + \delta x = x + \delta_1 x + \delta_2 x + \ldots\ldots + \delta_{\mathrm{n}} x = x + \Sigma \delta_m x. \qquad (3)$$

Then R is to be developed in terms of R and these variables, which represent the perturbations of the six elements.

Let v be taken as the general representative of the three elements which have been denoted by x, y, z, and I for the representative of the corresponding coefficients F, G, H; and similarly suppose V to represent the elements which have been denoted by

X, Y, Z and i to represent the corresponding coefficients f, g, h. Then it may be supposed that

$$\mathrm{v} = v + \delta v = v + \Sigma \delta_m v, \qquad \mathrm{V} = V + \delta V = V + \Sigma \delta_m V, \tag{4}$$

where δv, δV represent the periodical perturbations of the elements whose mean values are represented by v and V. Assume further that

$$\begin{aligned} v^{-1}\delta v &= \Sigma c_p \cos \gamma_p, & \delta V &= \Sigma \mathrm{C}_p \sin \gamma_p, \\ v^{-1}\delta_m v &= \Sigma j_p^m \frac{\mathrm{I}}{w_p} R_p^m \cos w_p, & \delta_m V &= \Sigma \mathrm{J}_p^m \frac{\mathrm{I}}{w_p} R_p^m \sin w_p, \end{aligned} \tag{5}$$

where m denotes the number of accents to be attached to j and J.

Next by successive direct differentiation

$$v^s \frac{d^s R}{dv^s} = \Sigma \frac{\Gamma(\mathrm{I})}{\Gamma(\mathrm{I}-s)} R \cos w, \qquad \frac{d^s R}{dV^s} = \Sigma i^s \{\cos s\tfrac{\pi}{2} . R \cos w - \sin s\tfrac{\pi}{2} . R \sin w\}. \tag{6}$$

Put

$$\mathrm{D}_v = [v^{-1}\delta v]\, v \frac{d}{dv}, \qquad \mathrm{D}_V = \delta V \frac{d}{dV},$$

then a known form of expansion gives the expression

$$\begin{aligned} \mathrm{R} = R + \delta R &= \{\epsilon^{\Sigma(\mathrm{D}_v + \mathrm{D}_V)}\} R \\ &= \{\Sigma(\mathrm{I} + \mathrm{D}_v + \tfrac{\mathrm{I}}{\underline{2}}[\mathrm{D}_v]^2 + \dots)(\mathrm{I} + \mathrm{D}_V + \tfrac{\mathrm{I}}{\underline{2}}[\mathrm{D}_V]^2 + \dots)\dots\} R. \end{aligned} \tag{7}$$

From the value assigned to D the following expressions can be derived:—

$$\begin{aligned} [\mathrm{D}_v]^n R \cos w &= \left(\frac{\mathrm{I}}{2}\right)^n \{\mathrm{I}(\mathrm{I}-1)\dots(\mathrm{I}-n+1)\}[c_1 + c_2 + c_3 + \dots + c_n]^n \\ &\qquad \cos(w + q'\gamma_1 + q''\gamma_2 + \dots + q^n\gamma_n), \\ [\mathrm{D}_V]^n R \cos w &= \left(\frac{\mathrm{I}}{2}\right)^n \{q'q''\dots q^n\}(i)^n \qquad [\mathrm{C}_1 + \mathrm{C}_2 + \mathrm{C}_3 + \dots + \mathrm{C}_n]^n \\ &\qquad \cos(w + q'\gamma_1 + q''\gamma_2 + \dots + q^n\gamma_n), \end{aligned} \tag{8}$$

$$[\mathrm{D}_v]^m [\mathrm{D}_V]^{m-n} R \cos w = \left(\frac{\mathrm{I}}{2}\right)^n \{q^m q^{m+1}\dots q^n\}(i)^{n-m}\{\mathrm{I}(\mathrm{I}-1)\dots(\mathrm{I}-m+1)\}$$
$$[c_1 + c_2 + \dots + c_n]^m [\mathrm{C}_m + \mathrm{C}_{m+1} + \dots + \mathrm{C}_n]^{n-m} \cos(w + q'\gamma_1 + q''\gamma_2 + \dots + q^n\gamma_n).$$

Then putting for brevity

$$\begin{aligned} (\mathrm{I}c)^m &= \{\mathrm{I}(\mathrm{I}-1)\dots(\mathrm{I}-m+1)\}[c_1 + c_2 + \dots + c_m]^m, \\ (qi\mathrm{C})^m &= (i)^m\{q'q''\dots q^m\} \qquad [\mathrm{C}_1 + \mathrm{C}_2 + \dots + \mathrm{C}_m]^m, \end{aligned} \tag{9}$$

the complete value of the general term of R will be

$$\mathrm{R}\cos \mathrm{w} = R\cos w + \frac{1}{2} R\,(\mathrm{I}c_1 + q'i\mathrm{C}_1)\cos(w + q'\gamma_1) + \ldots$$
$$+ \frac{1}{\underline{n}}\left(\frac{1}{2}\right)^n R\,(\mathrm{I}c + qi\mathrm{C})^n \cos(w + \Sigma_1{}^n q^m \gamma_m) + \ldots \quad (10)$$

In the preceding expressions the coefficients $q'\ q''\ \ldots\ q^m$ only take the values plus or minus unity, and have been introduced for the purpose of clearly showing the changes of sign produced by changes of sign in the argument.

Let

$$\mathrm{R} = \Sigma\{R_p + \mathrm{A}_p\}\cos w_p, \quad (11)$$

where A_p denotes the sum of the coefficients of the argument w_p which arises from the portion of R depending on the perturbations, so that $R_p + \mathrm{A}_p$ denotes the complete value of the coefficient of the argument w_p in the value of R. The function A is therefore a constant, and a function of R_p, the coefficients I, i, q, and the indeterminate coefficients c and C.

Let (R) be a function of similar form to R, and such that

$$(\mathrm{R}) = \mathrm{x}^{(\mathrm{F})}\mathrm{y}^{(\mathrm{G})}\mathrm{z}^{(\mathrm{H})}\,\mathrm{R}, \quad (12)$$

so that the general term of (R) may be written in the form

$$(\mathrm{R})\cos \mathrm{w} = (\Lambda)\,\mathrm{x}^{\mathrm{F}+(\mathrm{F})}\mathrm{y}^{\mathrm{G}+(\mathrm{G})}\mathrm{z}^{\mathrm{H}+(\mathrm{H})}\cos\{(\lambda) + f\mathrm{X} + g\mathrm{Y} + h\mathrm{Z}\}, \quad (13)$$

and is evidently merely a particular case of the general term (1). Its expansion will be identical in form with that of (10). Suppose therefore,

$$(\mathrm{R}) = \Sigma x^{(\mathrm{F})} y^{(\mathrm{G})} z^{(\mathrm{H})}\{R_p + (\mathrm{A})_p\}\cos w_p, \quad (14)$$

where $(\mathrm{A})_p$ only differs from A_p in F, G, H being replaced by F+(F), G+(G), H+(H) respectively; so that $(\mathrm{A})_p$ is merely a particular case of A_p.

Suppose the coefficients of the perturbations in the six elements of the orbit to be represented by the $6k$ indeterminate coefficients $c_1\ \ldots\ c_{6k}$, then the function A_p can be written in the form

$$\mathrm{A}_p = \Sigma\phi(r)\,c_s + \Sigma\phi'(r)\,(c_s c_{s_1}) + \Sigma\phi''(r)\,(c_s c_{s_1} c_{s_2}) + \ldots + \Sigma\phi^n(r)\,(c_s \ldots c_{s_n}) + \ldots, \quad (15)$$

where $\phi(r), \ldots \phi^n(r)$ are known constants, functions of R_r, I_r, i_r, and $c_s, \ldots c_{s_n}$ are particular cases of the series of indeterminate coefficients which represent the perturbations of the elements. The summation extends to all the combinations of the suffixes r and $s, \ldots s_n$ which will yield a term with the argument w_p.

By means of (10) the value of the coefficient A_p as developed in (15) can be written down to the nth order. It is merely

necessary to know the different combinations of the arguments $w_r, \gamma_1, \ldots \gamma_k$ which will render $w_r + q\gamma_1 + \ldots + q^m\gamma_m$ equal to the value w_p. Suppose there are l such independent combinations of $w_r, \gamma_1, \ldots \gamma_k$, then by giving to $s, \ldots s_m$ and r these l series of values, the complete value of $\Sigma\phi^m(r)\,(c_s \ldots c_{s_m})$ as it exists in A_p will be obtained. It only remains therefore to determine these l combinations of the arguments. For this purpose let

$$R = \Sigma_0^k R_m \cos w_m.$$

By direct successive multiplication form the functions

$$\{R\}^2,\ \{R\}^3, \ldots \{R\}^n,$$

neglecting terms of a higher order than necessary for an approximation to the given order. Then the different combinations of m in the value of the coefficient of the argument w_p in the function $\{R\}^m$ will be the l combinations of $r, s_1, \ldots s_n$ in the value of $\Sigma\phi^m(r)\,(c_s \ldots c_{s_m})$ as it exists in the coefficient A_p.

In this way the value of A_p can be completely determined to the nth order, or the $2n$th order as usually computed, so that R may be regarded as completely determined to that order.

Suppose the system of elements represented by v, V to be a canonical system, so that the differential equations which give the perturbations in the elements may be written in the form

$$\frac{dv}{dt} = \frac{dR}{dV} = -\Sigma i R \sin w, \qquad \frac{dV}{dt} = \frac{dR}{dv} = v^{-1}\Sigma I R \cos w. \tag{16}$$

Let it be assumed that

$$\int \begin{matrix}\cos\\ \sin\end{matrix}(w)\,dt = \pm\frac{1}{w}\begin{matrix}\sin\\ \cos\end{matrix}(w),$$

and integrate the system of equations (16). They become

$$v^{-1}\delta v = \Sigma\frac{i}{v}\frac{1}{w}\{R + \delta R\}\cos w, \qquad \delta V = \Sigma\frac{I}{v}\frac{1}{w}\{R + (\delta R)\sin w\}. \tag{17}$$

Therefore

$$\begin{aligned} c_p \cos w_p &= \{i_p\frac{I}{v}\frac{I}{w_p}R_p + i_r\frac{I}{v}\frac{I}{w_p}A_p\}\cos w_p, \\ C_p \sin w_p &= \{I_p\frac{I}{v}\frac{I}{w_p}R_p + I_r\frac{I}{v}\frac{I}{w_p}(A)_p\}\sin w_p. \end{aligned} \tag{18}$$

In this manner, amongst the $6k$ coefficients $c_1, \ldots c_{6k}$, there will be obtained $6k$ independent equations of the form

$$c_p = \frac{I}{v}\frac{I}{w_p}\{i_p R_p + \Sigma i_r\,\phi'(r)\,c_s + \ldots + \Sigma i_r\,\phi^n(r)\,(c_s \ldots c_{s_n}) + \ldots\}, \tag{19}$$

where, for brevity, I is included under i and C under c.

This system of $6k$ equations amongst the $6k$ indeterminate coefficients can be completely solved without difficulty and the values of the $6k$ coefficients $c_1, \ldots c_{6k}$ can be determined to any required order.

It would not be difficult to transform the preceding equations into others of a still more advantageous form, but this is unnecessary at present. In practice it would be found essential to have seven series of unknown coefficients, the extra one being required for the perturbations of the mean motion. Therefore, although the preceding system of equations is very simple, and can be written down without difficulty to any required order, it still retains the serious disadvantage of requiring seven variables. When indeterminate coefficients are employed, this multiplicity of variables is the great difficulty of the method of perturbed elements, for it involves the solution of a system of $7k$ equations amongst $7k$ unknown quantities, whereas, when the coordinates are made the direct variables, as in my "General Method of Treating the Lunar Theory," it is only necessary to solve a system of $3k$ equations amongst $3k$ unknown quantities, and this requires far less labour. Moreover, the preceding method requires the entire system of perturbations to be attacked at once, and it is not practicable to determine the value of any one coefficient by itself. It remains therefore to remove these disadvantages by further developing the method.

Instead of supposing R to be developed in terms of the variables represented by δv and δV, let it be developed in terms of the n times as many variables represented by $\delta_1 v, \ldots \delta_n v$ and $\delta_1 V, \ldots \delta_n V$, and denoted generally by $\delta_m v$ and $\delta_m V$. As the functions represented by $\delta_m v$ and $\delta_m V$ are perfectly analogous to those represented by δv and δV, the form of the expansion will not be altered, and it will merely be necessary to make some slight changes in the notation. Put, then,

$$\left[R_s^m (\mathrm{I}_r j_s^m) \frac{\mathrm{I}}{w_s}\right]^k = \{\mathrm{I}_r (\mathrm{I}_r - \mathrm{I}) \ldots (\mathrm{I}_r - k + \mathrm{I})\} \left(j_{s_1}^m \mathrm{R}_{s_1}^m \frac{\mathrm{I}}{w_{s_1}} + \ldots + j_{s_k}^m R_{s_k}^m \frac{\mathrm{I}}{w_{s_k}}\right)^k \tag{20}$$

$$\left[R_s^m (q^i{}_r \mathrm{J}_s^m) \frac{\mathrm{I}}{w_s}\right]^k = \{q' q'' \ldots q^k\} (i)^k \left(\mathrm{J}_{s_1}^m R_{s_1}^m \frac{\mathrm{I}}{w_{s_1}} + \ldots + \mathrm{J}_{s_k}^m R_{s_k}^m \frac{\mathrm{I}}{w_{s_k}}\right)^k$$

and let Σ denote a summation with respect to m, so that, for example,

$$\left[\Sigma R_s^m (\mathrm{I}_r j_s^m) \frac{\mathrm{I}}{w_s}\right]^k = \{\mathrm{I}_r (\mathrm{I}_r - \mathrm{I}) \ldots (\mathrm{I}_r - k + \mathrm{I})\}$$

$$\left([j'_{s_1} R'_{s_1} + \ldots + j_{s_1}^n R_{s_1}^n] \frac{\mathrm{I}}{w_{s_1}} \ldots + [j'_{s_k} R'_{s_k} + \ldots + j_{s_k}^n R_{s_k}^n] \frac{\mathrm{I}}{w_{s_k}}\right)^k \tag{21}$$

Then the general term of R can be written in the form

$$\mathrm{R}\cos \mathrm{w} = R_r \cos w_r + \frac{\mathrm{I}}{2}\left[\Sigma R_{s_1}^m \left(\mathrm{I}_r j_{s_1}^m + q' i_r \mathrm{J}_{s_1}^m\right) \frac{\mathrm{I}}{w_{s_1}}\right] R_r \cos\left(w_r + q' w_{s_1}\right) + \ldots$$

$$+ \frac{\mathrm{I}}{\lfloor n}\left(\frac{\mathrm{I}}{2}\right)^n \left[\Sigma R_s^m \left(\mathrm{I}_r j_s^m + q i_r \mathrm{J}_s^m\right) \frac{\mathrm{I}}{w_s}\right]^n R_r \cos\left(w_r + \Sigma_1^n q w_s\right) + \ldots \qquad (22)$$

Substitute this value with that of (R) in the differential equations, and on integration they become

$$v^{-1}\delta v = \frac{\mathrm{I}}{v} i_r \frac{\mathrm{I}}{w_r} R_r \cos w_r + \ldots + \frac{\mathrm{I}}{v} i_r \frac{\mathrm{I}}{w_r + \Sigma_1^n q w_s} \times$$

$$\frac{\mathrm{I}}{\lfloor n}\left(\frac{\mathrm{I}}{2}\right)^n \left[\Sigma R_s^m \left(\mathrm{I}_r j_s^n + q i_r \mathrm{J}_s^m\right) \qquad \frac{\mathrm{I}}{w_s}\right]^n R_r \cos\left(w_r + \Sigma_1^n q w_s\right) + \ldots \qquad (23)$$

$$\delta V = \frac{\mathrm{I}}{v} \mathrm{I}_r \frac{\mathrm{I}}{w_r} R_r \sin w_r + \ldots + \frac{\mathrm{I}}{v} \mathrm{I}_r \frac{\mathrm{I}}{w_r + \Sigma_1^n q w_s} \times$$

$$\frac{\mathrm{I}}{\lfloor n}\left(\frac{\mathrm{I}}{2}\right)^n \left[\Sigma R_s^m \left(\mathrm{I}_r j_s^m + q i_r \mathrm{J}_s^m + (\mathrm{I})\, j_s^m\right) \frac{\mathrm{I}}{w_s}\right]^n R_r \sin\left(w_r + \Sigma_1^n q w_s\right) + \ldots$$

where, as before, the transformation of R into (R) is effected by the addition of the quantity (I), where (I) is an absolutely constant quantity, which is the same for all values of r and s.

Assume that the perturbations of the elements of the order of the mth power of the disturbing forces are represented by the expressions

$$v^{-1}\delta_m v = j^m \frac{\mathrm{I}}{w} R^m \cos w, \qquad \delta_m V = \mathrm{J}^m \frac{\mathrm{I}}{w} R^m \sin w, \qquad (24)$$

then it only remains to determine the value of the coefficients $j' \ldots j^n$ and $\mathrm{J}' \ldots \mathrm{J}^n$ and the quantities $R' \ldots R^n$, and the right hand side of the equations (23) will become an explicit function of known quantities, so that the complete value of the perturbations of the elements will have been determined to the nth order of the disturbing forces.

Neglecting the square of the disturbing forces, the equations (22) can be written in the form

$$v^{-1}\delta_1 v = \frac{\mathrm{I}}{v} i_p \frac{\mathrm{I}}{w_p} R_p \cos w_p, \qquad \delta_1 V = \frac{\mathrm{I}}{v} \mathrm{I}_p \frac{\mathrm{I}}{w_p} R_p \sin w_p, \qquad (25)$$

where the suffix r has been replaced by p. Comparing with (24),

$$j'_p = \frac{\mathrm{I}}{v} i_p, \qquad \mathrm{J}'_p = \frac{\mathrm{I}}{v} \mathrm{I}_p, \qquad R'_p = R_p. \qquad (26)$$

Thus all the terms depending on j', J', R' may be regarded as known.

As far as these terms are concerned, the complete value of the term in the disturbing function which has the argument w_k may be written in the form

$$\mathrm{R}\cos\mathrm{w} = R_k \cos w_k + \left[\frac{\mathrm{I}}{2} R_{s_1} (\mathrm{I}_r i_{s_1} + q' i_r \mathrm{I}_{s_1}) \frac{\mathrm{I}}{v} \frac{\mathrm{I}}{w_{s_1}}\right] R_r \cos (w_r + q' w_{s_1}) + \ldots$$

$$+ \frac{\mathrm{I}}{\underline{n}} \left[\frac{\mathrm{I}}{2} R_s (\mathrm{I}_r i_s + q i_r \mathrm{J}_s) \frac{\mathrm{I}}{v} \frac{\mathrm{I}}{w_s}\right]^n R_r \cos (w_r + \Sigma_1^n q w_s) + \ldots \qquad (27)$$

with the single condition that the suffixes r, s_1, s_2, . . . s_n which have been used to distinguish quantities belonging to different arguments are to be given in turn all the different combinations of values which will make $w_r + q' w_s + \ldots + q^n w_{s_n}$ equal to w_k. These values will be identical with the l different combinations of the suffixes m in the value of the coefficient of the term with the argument w_k in the function $\{R\}^n$ or nth power of R.

The coefficients $j'', \ldots j^n$ and $\mathrm{J}'', \ldots \mathrm{J}^n$ can be determined in exactly the same manner, and will give rise to similar expression, the principal difference being that the simple coefficients i_s, I_s will be replaced by more complex functions. For the present it is unnecessary to write down these terms seriatim, although it can be at once done from equations (23). The general expression for the terms of the perturbations of the elements of the nth order of the disturbing forces will be

$$v^{-1} \delta_n v = \Sigma j'_r \frac{\mathrm{I}}{w_p} R_r \frac{\mathrm{I}}{\underline{b_1} \ldots \underline{b_m}}$$

$$\left\{\left[\frac{\mathrm{I}}{2} R_s (\mathrm{I}_r j'_s + q i_r \mathrm{J}'_s) \frac{\mathrm{I}}{w_s}\right]^{b_1} \ldots \left[\frac{\mathrm{I}}{2} R_s (\mathrm{I}_r j_s^m + q i_r \mathrm{J}_s^m) \frac{\mathrm{I}}{w_s}\right]^{b_m}\right\} \cos (w_r + \Sigma_1^b q w_s)$$

$$\delta_n V = \Sigma \mathrm{J}'_r \frac{\mathrm{I}}{w_p} R_r \frac{\mathrm{I}}{\underline{b_1} \ldots \underline{b_m}} \qquad (28)$$

$$\left\{\left[\frac{\mathrm{I}}{2} R_s (\mathrm{I}_r j'_s + q i_r \mathrm{J}_s + [\mathrm{I}] j'_s) \frac{\mathrm{I}}{w_s}\right]^{b_1} \ldots \left[\frac{\mathrm{I}}{2} R_s (\mathrm{I}_r j_s^m + q i_r \mathrm{J}_s^m + [\mathrm{I}] j_s^m) \frac{\mathrm{I}}{w_s}\right]^{b_m}\right\} \sin (w_r + \Sigma_1^b q w_s),$$

with the conditions that the constants are to take in succession all values which satisfy the equations

$$\begin{aligned} w_r + q' w_{s_1} + q'' w_{s_2} + \ldots + q^b w_{s_b} &= w_p \\ b_1 + b_2 + b_3 + \ldots + b_m &= b \\ b_1 + 2b_2 + 3b_3 + \ldots + m b_m &= n - \mathrm{I}. \end{aligned} \qquad (29)$$

These expressions may seem to be complicated, but this is inseparable from the terms being given for the nth order of the dis-

turbing forces. As far as the fifth power of the disturbing forces or m^{11} of the ordinary notation, they may be written down in an extremely simple form, and even to the tenth order, or m^{21}, they are neither complicated nor lengthy. The different combinations which can be taken by the suffixes r, s_1, ... s_n will be identical with the combinations of the suffixes m in the value of the coefficient of the term with the argument w_p, in the function $\{R\}^b$ or bth power of R.

In this manner, therefore, the complete value of the perturbations of the elements of the orbit may be determined to the nth order of the disturbing forces, and this forms a complete solution of the problem of disturbed elliptic motion. The expressions which have been obtained for the perturbations admit of being transformed into still more advantageous forms. It is necessary, however, to defer these extensions and developments to a place in a second and more elaborate communication. It may be mentioned, however, that by these transformations the complete value of the perturbations of the elements may be made to assume the following form:—

$$v^{-1}\delta v = \Big([\mathrm{P}]^p_1 R_p + \{[\mathrm{P}]^r_1 [\mathrm{P}']^r_{s_1}\} R_r R_{s_1} + \ldots + \{[\mathrm{P}]^r_1 ([\mathrm{P}^n]^r_{s_1} + \ldots + [\mathrm{P}^n]^r_{s_n})^n\} (R_r R_{s_1} \ldots R_{s_n}) + \ldots\Big) \cos w_p$$

$$\delta V = \Big([\mathrm{Q}]^p_1 R_p + \{[\mathrm{Q}]^p_1 [\mathrm{Q}']^r_{s_1}\} R_r R_{s_1} + \ldots + \{[\mathrm{Q}]^r_1 ([\mathrm{Q}^n]^r_{s_1} + \ldots + [\mathrm{Q}^n]^r_{s_n})^n\} (R_r R_{s_1} \ldots R_{s_n}) + \ldots\Big) \sin w_p. \qquad (30)$$

In this expression R_p, R_r, R_{s_1}, ... R_{s_n} are the coefficients of the arguments w_p, w_r, w_{s_1} ... w_{s_n} in the ordinary disturbing function. The functions denoted by $[\mathrm{P}]^r_{s_m}$ and $[\mathrm{Q}]^r_{s_m}$ are known functions of the coefficients I_r, I_{s_m}, i_r, i_{s_m} ... and the integrating factors w_{s_m} ... They may be transformed into explicit functions of the form

$$\phi\left\{(\mathrm{I}_r i_{s_m} + q i_r \mathrm{I}_{s_m}) \frac{\mathrm{I}}{v} \frac{\mathrm{I}}{w_{s_m}}\right\} = [\mathrm{P}]^r_{s_m}$$

$$\psi\left\{(\mathrm{I}_r i_{s_m} + q i_r \mathrm{I}_{s_m} + (\mathrm{I})\, i_{s_m}) \frac{\mathrm{I}}{v} \frac{\mathrm{I}}{w_{s_m}}\right\} = [\mathrm{Q}]^r_{s_m},$$

only under this form the terms depending on high powers of the disturbing forces are rendered more complex and lengthy.

No restriction has been placed on the number of variables represented by v and V. In the expression for the general term of R it has been assumed that they will be only six in number, or x, y, z, X, Y, Z. These are the number required by the lunar theory. The method can be applied without alteration to the planetary theory, when there will be twelve variables,

x, x', y, y', z, z', X, X', Y, Y', Z, Z', the accented quantities referring to the disturbing planet. It can be applied, however, to any number of such variables, and absolutely without alteration.

In the preceding investigation, instead of employing the ordinary elliptic elements, use has been made of the six undefined elements whose mean values are x, y, z, X, Y, Z, and no restriction has been placed on them beyond the single condition that they are to be such as will yield a canonical system of differential equations. The perturbations of these elements have been completely determined in explicit series of functions of the form

$$R_r = (\Lambda) x^{\mathrm{F}_r} y^{\mathrm{G}_r} z^{\mathrm{H}_r},$$

$$w_s = (\lambda) + f_s X + g_s Y + h_s Z, \tag{31}$$

$$(\mathrm{I}_r i_s + q i_r \mathrm{I}_s)\frac{\mathrm{I}}{v} = (\mathrm{F}_r f_s + q \mathrm{F}_s f_r)\frac{\mathrm{I}}{x} + (\mathrm{G}_r g_s + q \mathrm{G}_s g_r)\frac{\mathrm{I}}{y} + (\mathrm{H}_r h_s + q \mathrm{H}_s h_r)\frac{\mathrm{I}}{z}.$$

The value of the disturbing function has been already determined by me to the ninth order in terms of the usual elements of the elliptic orbit. Two courses can be adopted to render this result available for the purpose of the present method; either it can be transformed into an expression in terms of the six elements, x, y, z, X, Y, Z, or, *vice versâ*, the expression (31) can be transformed into terms of the usual elements.

Suppose

$$X + Y + Z = \mathrm{L} + \mathrm{A} + \mathrm{B} + \text{constant} = w_r,$$

where L, A, B denote the mean longitude of the Moon and the position of the perigee and ascending node of the lunar orbit. Then, as long as this condition is maintained, the arguments of the terms in the disturbing function will remain unaltered, with the exception that the coefficients f_r, g_r, h_r will be transformed into other particular values f_k, g_k, h_k.

Similarly the coefficient of the term with the argument w_r in the ordinary disturbing function may be written in the form

$$b_1 a^{\mathrm{F}_k} e^{\mathrm{G}_k} \eta^{\mathrm{H}_k} (1 + b_2 a^2 + b_3 e^2 + b_4 \eta^2 + \ldots),$$

where F_k, G_k, H_k are numerical constants, and assuming

$$x = \phi_1(a, e, \eta), \qquad y = \phi_2(a, e, \eta), \qquad z = \phi_3(a, e, \eta),$$

the preceding may be transformed into the value

$$c_1 x^{\mathrm{F}_r} y^{\mathrm{G}_r} z^{\mathrm{H}_r} (1 + c_2 x^2 + c_3 y^2 + c_4 z^2 + \ldots);$$

the only difference being that F_r, G_r, H_r will be different integers to F_k, G_k, H_k.

It is evident, therefore, that by properly defining x, y, z, either the usual disturbing function can be transformed into

explicit functions of X, Y, Z, x, y, z, or the equations (31) transformed into explicit functions a, e, η, L, A, B, and their coefficients of F_k, G_k, H_k, f_k, g_k, h_k.

It may be remarked that the preceding method has been already employed to deduce some remarkably interesting and important theorems with regard to the terms of long period in the mean longitude. From their form it is obvious that any single term or set of terms can be determined by themselves, so that the method is peculiarly well adapted for determining the terms of long period which converge slowly and require the fourth and fifth powers of the disturbing forces to be taken into consideration. In a subsequent communication I hope to be able to deduce by its aid some important theorems with regard to the secular inequalities in the motion of the planets depending on the third and fifth powers of the disturbing forces.

London,
December 9, 1878.

Note on the Presence of Particles of Meteoric Dust in the Atmosphere.

By A. C. Ranyard, Esq.

In a Paper* read before the British Association in 1852, Professor Andrews announced that he had discovered particles of native iron in the basalt of the Giant's Causeway. Having reduced portions of the rock in a porcelain mortar to a tolerably fine powder, magnetic particles were collected by passing a magnet several times through the powder. The particles adhering to the magnet were then placed under the microscope and moistened with an acid solution of sulphate of copper. On some of them copper was deposited in a manner which indicated the presence of native iron. Professor Andrews appears to have suggested that the particles of native iron may have been derived from meteors which fell when the basalt was in a plastic condition.

In 1867 Dr. T. L. Phipson published a book entitled *Meteors, Aerolites, and Falling Stars*, in which he states† that he had frequently exposed to the wind a sheet of glass covered with some transparent mucilaginous substance in order to catch the particles of dust floating in the air. He says: "I have found that when a glass covered with pure glycerine is exposed to a strong wind late in November, it receives a certain number of *black angular particles*, some three or four of which may be thus collected in the space of a couple of hours. The experiment being made far in the country, away from the 'smuts' of a town, the black particles show themselves all the same. They are, however, not soot or charcoal; they can be dissolved in strong

* *Brit. Assoc. Reports* for 1852. Part II. pp. 34, 35. † *See* pp. 229, 230.

hydrochloric acid, and produce yellow chloride of *iron* upon the glass plate." He continues: "Although I have made this experiment at various periods of the year, and in different countries, it is only in the winter months that the black particles, giving with hydrochloric acid chloride of *iron*, have been met with."

Towards the end of 1871 Dr. Nordenskjöld collected some apparently pure snow which fell in the neighbourhood of Stockholm during a heavy snowstorm.* On melting a cubic metre of the snow collected, towards the end of the fall, he found that it left a black residue, from which he was able to extract with a magnet particles which, when rubbed in an agate mortar, exhibited metallic characters, and on being treated with acid proved to be iron.

In 1872 Dr. Nordenskjöld obtained some snow from off the ice of the Rantajerwi, at a spot which is separated by a dense forest from the nearest houses at Evoia, in Finland. When melted the snow yielded a soot-like residue, which under the microscope was found to consist of white or yellowish white granules, with a black carbonaceous substance, from which the magnet removed black grains, which when rubbed in a mortar were seen to be iron.

The Arctic Expedition of 1872 presented an opportunity for the collection of snow in a region removed as far as possible from human habitations. On August 8 the snow covering the drift ice, at lat. 80° N. and long. 13° E., was observed to be thickly covered with small black particles, while in places these penetrated to a depth of some inches the granular mass of ice into which the underlying snow had been converted. Magnetic particles were abundant, and their power to reduce copper sulphate was established. Again, on September 2, in lat. 80° N., long. 15° E., the ice-field was found covered with a bed of freshly fallen snow 50 mm. thick, then a more compact bed 8 mm. in thickness, and below this a layer 30 mm. thick of snow converted into a crystalline granular mass. The latter was full of black granules, which became grey when dried, and exhibited the magnetic and chemical characters already referred to; they amounted to 0·1 to 1·0 millegramme in a cubic metre of snow. Analysis of some millegrammes enabled Dr. Nordenskjöld to establish the presence of iron, phosphorus, cobalt, and probably nickel.

During the years 1874, 1875, and 1876 M. Tissandier published in the *Comptes Rendus*,† an interesting series of papers on his examination of atmospheric dust. He showed that in the dust deposited upon the towers of the Church of Notre Dame, as well as in the solid matter deposited from rain-water, there were metallic particles containing iron, nickel, and cobalt. On examining these particles under the microscope, he found that they were very similar in appearance to particles which he was

* *See Comptes Rendus*, lxxvii., 463.

† *See* the Numbers for March 28, 1874; January 4, 1875; October 4, 1875; and July 3, 1876.

able to detach by friction from the surface of meteorites, and he concludes that they are the solidified metallic rain detached from meteoric masses during their passage through the atmosphere.

Dr. Walter Flight published, in the *Geological Magazine* for March and April 1875, an important paper upon meteoric dust, which has since been reprinted in the *Arctic Manual.* After referring to Dr. Nordenskjöld's observations, he remarks that the dust from the polar ice north of Spitzbergen bears a great resemblance to a substance termed *cryoconite* "which was found in Greenland in 1870 very evenly distributed, in not inconsiderable quantity, on shore ice, as well as on ice thirty miles from the coast and at a height of 700 metres above the sea. The dust of both localities has probably a common origin. The cryoconite is chiefly met with in the holes of the ice, forming a layer of grey powder at the bottom of the water filling the holes. Considerable quantities of this substance are often carried down by streams which traverse the glacier in all directions. The ice hills which feed these streams lie towards the east, on a slowly rising, undulating plateau, on the surface of which not the slightest trace of stone or larger rock masses was observed. The actual position of this material, to which Dr. Nordenskjöld has given the name of cryoconite (κρύος ice, and κόνις dust), in open hollows on the surface of the glacier, precluded the possibility of its having been derived from the ground beneath."

After describing the chemical composition of cryoconite, Dr. Flight continues: "The origin of cryoconite is highly enigmatical. That it is not a product of the weathering of the gneiss of the coast is shown by its inferior hardness, indicating the absence of quartz, the large proportion of soda, and the fact of mica not being present. That it is not dust derived from the basalt area of Greenland is indicated by the subordinate position iron-oxide occupies among the constituents, as well as by the large proportion of silicic acid. We have then to fall back on the assumption that it is either of volcanic or cosmical origin." . . . . "The cryoconite, whencesoever it comes, contains one constituent of cosmical origin. Dr. Nordenskjöld extracted, by means of the magnet, from a large quantity of material sufficient particles to determine their metallic nature and composition. These grains separate copper from a solution of the sulphate and exhibit conclusive indications of the presence of cobalt (not only before the blowpipe, but with solution of potassium-nitrite), of copper, and of nickel, though in the latter case with a smaller degree of certainty, through the reactions of this metal being of a less delicate character."

In 1876 Mr. John Murray published a paper in the *Proceedings of the Royal Society of Edinburgh,** in which he gave an account of his examination of the deposits found at the bottom of the oceans and seas visited by H.M.S. *Challenger.* In many of

* Vol ix., pp. 247-262.

the deep sea clays Mr. Murray found numerous magnetic particles, some of which he extracted by means of a magnet carefully covered with paper. On placing them under a microscope and moistening with the acid solution of sulphate of copper, he found that copper was deposited on some of the particles. From this and the circumstance that the particles bore a strong resemblance to particles found on the "mammillated outer surface of the Cape Meteorite," Mr. Murray concluded that the particles had a cosmic origin.

He suggests that the reason meteoric particles are found in such abundance in the deep sea clays is that at the bottom of the ocean far from land such particles would not be washed away or so rapidly covered up as in the case of deposits formed nearer to continents, and they would consequently appear to form a larger proportion of the deposited matter. He also suggests that the nickel present in meteoric iron would greatly retard the oxidation of such particles. Professor Alexander Herschel has, I understand, examined under the microscope some of the particles extracted by Mr. Murray, and concurs with him in the opinion that they probably have a cosmic origin.

In September 1876 Mons. E. Yung published a paper* entitled "Étude sur les Poussières cosmiques." He gives a plate showing iron particles which he had found in snow that fell at the Hospice of St. Bernard. During the years 1875 and 1876 he examined snow which fell on several other Swiss mountains, and in every instance found iron particles. He also extracted, by means of a magnet, globules of iron from dust collected upon the towers of churches. The iron particles which he figures in his plate are mostly spherical or tear-shaped, with projecting points and threads of metal. His observations entirely confirm those of M. Tissandier.

The above observations seem to point to a conclusion which has, I believe, been advocated for some time past by Mr. Proctor, viz. that meteoric matter is continually falling in quantities which, in the lapse of ages, must accumulate so as materially to contribute to the matter of the Earth's crust.† There can be little doubt that in the course of a year millions of meteors enter the Earth's atmosphere. A few of the larger masses reach the

* See *Bulletin de la Société Vaudoise des Sciences Naturelles.* Vol. xiv., pp. 494–506.

† The iron particles probably only form a very small part of the meteoric dust continually falling—for, of the larger masses which have been seen to fall, it has been estimated that not one in fifty is iron. Dr. Flight informs me that in the British Museum there are 202 stony meteorites, all of which have been seen to fall, and that there are only four iron meteorites which have been seen to fall. Stony meteorites consist for the most part of olivine, augite, hornblende, felspar, and other minerals, most of which are common in volcanic and metamorphic rocks, which therefore cannot be distinguished as having a meteoric origin unless they are found in masses. It is worthy of remark that all the elements which are common in meteorites are also common in the stratified rocks.

Earth's surface, but by far the greater number appear to be consumed in the higher atmosphere. The above observations show that minute particles of iron frequently reach the Earth's surface without having undergone any change such as might be expected to result from their passage through the air in an incandescent state. The subject is one of considerable interest, and I therefore give an account of a somewhat incomplete experiment which I made while crossing the Atlantic at the beginning of last September. When at a distance of about 1,000 miles from the American coast, I exposed some glass plates covered with glycerine to the wind. They were placed upon a wind vane behind a tin funnel which directed a current of air upon the centre of the plate. The wind vane was mounted near to the prow of the vessel, and during the time of the exposure the wind was blowing nearly at right angles to the course of the vessel.

Four plates were exposed for periods of 30 hours, 24 hours, 18 hours, and 20 hours respectively. Immediately after the exposure the plates were placed in a box such as is ordinarily used by photographers for carrying negatives, and the whole was wrapped in paper so as carefully to exclude dust till the plates could be brought to England for examination. When the box was again opened the plates were placed under the microscope, and, at Mr. Neison's suggestion, I treated them first with dilute hydrochloric acid and then with sulphocyanide of potassium, a process which would indicate the presence of iron particles by a bright red stain.

On the plate which was exposed for 18 hours a rather large particle containing iron was found. It was of a dark brown colour and was somewhat elongated, tapering slightly towards one end, but was not angular like the particles caught by Dr. Phipson. It was clearly visible to the naked eye, and I estimated it to be between the $\frac{1}{100}$ and the $\frac{1}{150}$ of an inch in its longest diameter. There were other traces of iron upon the plates, but only in very minute quantities, always in connection with minute hairs and cells which had lodged in the glycerine. I do not feel satisfied with the experiment; for although the plates were carefully cleaned and the glycerine made use of showed no traces of iron, the box in which the plates were carried had been lying about in Professor Henry Draper's laboratory in New York, and I omitted to make sure that it was perfectly free from dust before making use of it. On another occasion I would recommend that the box in which the plates are to be carried should be carefully cleaned and coated on the inside with glycerine. A box without a lock and with brass hinges should be made use of. It might also be worth while to vary the experiment by exposing a magnet to the wind with the poles covered with tinfoil. On removing the tinfoil the magnetic particles should be allowed to fall on a plate covered with glycerine, which could be kept for examination.

There can be little doubt that the air up to a great height above the Earth's surface is impregnated with dust. The sky as seen from the highest mountains appears of a dark blue, which indicates the presence of particles small compared with the wave length of light. It has usually been assumed that these particles must have been carried upwards from the Earth's surface by convection currents, but Professor Tyndall's experiments in Switzerland tend to show that the higher air is comparatively free from germs. I would suggest that the blue colour may be caused by dust derived from the débris of meteors, the smaller particles of which may possibly occupy months or even years in falling to the Earth's surface.

Much evidence has been collected by Professor von Niessl and others which tends to show that many of the larger meteoric masses enter the Earth's atmosphere with velocities which indicate that they are moving in hyperbolic orbits, and consequently do not belong to the solar system. It seems therefore probable that at all events a certain proportion of the meteoric dust is derived from sources outside the solar system. The Earth and planets, as they are carried along with the Sun in his motion through space, would thus receive a larger proportion of meteoric matter on their northern than on their southern hemispheres, and I would suggest, as a theory worthy of consideration, that this may account for the preponderating mass of the continents in the northern hemisphere of the Earth, and for the fact, which has so frequently been pointed out by physical geographers, that the great terrestrial peninsulas all taper towards the Southern Pole.*

The experiments of Professor Arthur Wright, of Yale, show that when meteoric masses are heated, considerable amounts of occluded gas are given off. We shall therefore, in considering the results which must follow from the continuous fall of meteoric matter, have to take into account the fact that gaseous matter is probably continually being added to the atmosphere. If the amount of gaseous matter taken from the air and stored up in a solid form by the agency of plants and animals, and by the oxi-

* The following facts with regard to the Moon and the planet *Mars* may also have some connection with the unequal addition of foreign matter on their northern and southern hemispheres. On the Moon the volcanic action has been decidedly more intense in the southern than in the northern hemisphere, and it will also be noticed that the great crater ranges run mostly north and south. On the planet *Mars*—if we adopt the delineation of the seas and continents given by Proctor in his map, which was chiefly made from the drawings of the planet by Dawes—there is, as on our Earth, a greater proportion of ocean surface in the southern than in the northern hemisphere. On *Mars* the land surface is decidedly greater than the ocean surface, so that the seas appear reduced to mere lakes and narrow inlets; but it will be noticed that these have their broadest expansion in the southern hemisphere, and that what has been termed the equatoreal girdle of continents has its medial line decidedly to the north of the Martial equator.

dation of mineral substances, does not counterbalance the amount continually being added to the atmosphere from meteors, together with the supplies derived from volcanic vents and from other sources from which the atmosphere may be recruited, it will be evident that the total amount of the atmosphere must either be increasing or decreasing. And the point to which I wish to draw attention is that such an increase or decrease would in time serve to account for great changes of temperature at the Earth's surface. If we suppose the Earth to pass through a region of space where there are comparatively few meteors, the height of the atmosphere would in the course of time be greatly decreased, and we should have a temperature at the sea level corresponding to the present temperature of our mountain-tops. In the language of geologists, a glacial epoch would be the result. If, on the other hand, the Earth passed through a region of space rich in meteors, containing occluded carbonic acid gas, the atmosphere would increase in depth, and a period like the carboniferous period might be the result, in which a semi-tropical vegetation might again flourish on the coast of Greenland.

Observations of the Transit of Mercury, May 6, 1878, *and of the Occultation of Mars by the Moon, June* 3, 1878, *made at the Glasgow Observatory.* By Professor R. Grant, Director of the Observatory.

Transit of Mercury.

Observed by Mr. Arthur Bowden, Principal Assistant. The instrument used was the Ochtertyre Equatoreal of 9 inches aperture, with a magnifying power of 240 applied. The external contact at ingress was lost. As the planet advanced upon the Sun's disk, the instant of the bisection of its disk by the Sun's limb was observed to occur at $3^h\ 12^m\ 31^s$ Greenwich Mean Time. The internal contact was well seen. It occurred at $3^h\ 14^m\ 15^s.7$ Greenwich Mean Time. A bright spot was distinctly seen on the disk of the planet a little before contact. It was well defined and appeared of the same colour as the Sun. When the planet had wholly entered upon the Sun the spot did not appear so bright, but as the Sun was all the time covered by a thin cirrus of constantly varying density, the difference in brightness of the spot may have been attributable to that cause. No indication of a ligament was perceptible at the time of the planet's entrance upon the Sun's disk, but immediately after internal contact the limb of the planet exhibited an elongation or distortion towards the Sun's limb. This appearance continued visible for about ten seconds after contact. The bright spot was seen only once after the

planet had advanced well on the Sun's disk. No appearance of a halo around the planet was visible, although the outline of the disk was carefully scanned, using in succession several magnifying powers.

Note by Professor Grant.—When the planet had wholly entered upon the Sun's disk I observed its appearance in the Ochtertyre Equatoreal. The disk was of a uniformly black colour and was exceedingly well defined. I failed to perceive any indication of the bright spot to which Mr. Bowden refers, nor could I discern the slightest trace of a nebulous ring around the planet.

Occultation of Mars by the Moon.

The phenomenon was observed by Professor Grant with the Ochtertyre Equatoreal, magnifying power 240. The final extinction of the planet's light occurred at $9^h\ 57^m\ 1^s$ Greenwich Mean Time. The reappearance was not seen.

Measures of the Diameter of Mercury made at the Princeton Observatory, May 6, 1878.

(*Communicated by Professor C. A. Young.*)

In a recent Number of the *Monthly Notices* (see vol. xxviii., p. 423) the result of a preliminary reduction of our observations was stated as $11''{\cdot}74$.

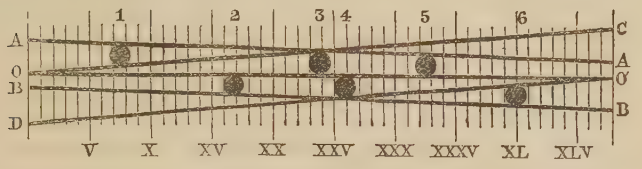
The rigorous reduction has changed this a little more than was anticipated, giving $11''{\cdot}705$ as the final result.

I had hoped to get a double-image micrometer in season for the Transit; but, failing to do so, adopted the method described below, which, so far as I know, is new and gives very satisfactory results.

The measures were made by bringing the image of the planet between two slightly converging bands or broad lines photographed on glass, and the observation consisted in noting the point where the disk exactly filled the interval between them. The bands were made nearly as wide as the planet, in order to neutralise irradiation. The plate upon which they were photographed was fitted into a micrometer in place of the usual spider-lines, and could be moved by the screw so as to bring any part of the plate to the centre of the field of view. The apparatus was attached to the Merz polarising eye-piece of the $9\frac{1}{2}$-inch Equatoreal, by which the intensity of the light could be graduated at pleasure. Powers of 150, 220, and 490 were used in the observations.

The arrangement of the reticule is indicated in the accompanying diagram, which, however, is not drawn to scale.

As will be seen, it affords six different points at which the measurements can be effected.



The plate was made as follows. A very careful drawing was constructed by Mr. Libbey, the base line OO′ being two feet long and accurately divided into half-inch spaces by a steel rule. This drawing was then photographed upon glass by Professor Brackett with a Dallmeyer rectilinear lens, which formed a reduced copy about half an inch long, distinct and sharp, and without the least distortion which could be detected by careful microscopic measurements made for the purpose. The slope of the bands A and B with reference to O was about 1 in 100, that of the bands C and D about 1 in 50.

The value of the graduated scale was ascertained by observing with the chronograph, on May 14, 16 transits of both limbs of the Sun over all the lines; the results were closely accordant and give—

1 div. (which is $0^{in}\cdot500$ on the original drawing) = $24''\cdot4996 \pm 0''\cdot0075$.

The distances between the bands at the points of measurement were determined in two ways: by direct measures of the glass plate under the microscope; and by measuring the original drawing with vernier callipers reading to thousandths of an inch. The two methods gave essentially the same result in every case; but the latter was preferred, as showing a smaller probable error.

The method pursued was as follows; taking, as an example, the measurement between A and O′ (No. 5 on the diagram):—

A moment's inspection of the record of observations shows that the mean result lies between 33 and 34 on the scale. Accordingly we measure, on the original paper drawing, the distance in thousandths of an inch between the adjacent edges of the bands A and O at the neighbouring graduation marks. The measurements were made by three different persons, and, the means being taken, we get the following numbers:—

At (31) the distance between A and O comes out $0^{in}\cdot2479$;
„ (32) „ „ „ $0^{in}\cdot2422$;
„ (33) „ „ „ $0^{in}\cdot2362$;
„ (34) „ „ „ $0^{in}\cdot2327$;
„ (35) „ „ „ $0^{in}\cdot2298$;
„ (36) „ „ „ $0^{in}\cdot2244$.

Each of these measurements gives an equation of condition of the form $x - ny = a$, when x is the corrected distance between A and O at (31), y is the change of distance corresponding to one division of the scale, and a is the measured distance at any scale division. Thus, for (36) the equation is $x - 57 = 0''.2244$.

The equations of condition are then solved by the method of least squares, and, with the values of x and y thus found, a corrected series of distances is constructed. These distances are reduced to seconds by the relation

$$\delta = \frac{D}{500} \times 24''.4996,$$

where δ is the distance in arc, and D the distance in thousandths of an inch.

The numbers are then tabulated, and with their help the result of each measurement of the planet is obtained by a simple interpolation at sight.

In the example cited the results are as follows :—

Series 5, A and O.

Scale Division.	Measured distance.	Corrected.	Value.
			"
31	247·9	246·8	12·09
32	242·2	242·3	11·87
33	236·2	237·8	11·65
34	232·7	233·3	11·43
35	229·8	228·7	11·21
36	224·4	224·2	10 99

The observations were much interrupted by flying clouds; hence the incompleteness of several of the series. It was intended to make 8 measurements at each point, 4 of the equatoreal and 4 of the polar diameter, but the fourth measurement of the polar diameter failed. The results of the six series are given below :—

(1) Between A and C.		(2) Between B and O.	
Equat. Diam.	Polar Diam.	Equat. Diam.	Polar Diam.
"	"	"	"
11·14	11·47	11·28	12·12
11·81	11·94	11·40	11·74
11·94	11·60	12·00	11·83
12·01		11·88	
Mean 11·73	11·67	Mean 11·64	11·90

(3) Between C and O.

Equat. Diam.	Polar Diam.
11·49	12·00
12·10	11·75
11·80	
11·64	
Mean 11·76	11·87

(4) Between D and O.

Equat. Diam.	Polar Diam.
11·81	11·91
12·02	11·40
11·40	
11·76	
Mean 11·75	11·66

(5) Between A and O.

Equat. Diam.	Polar Diam.
11·19	11·91
11·87	11·54
11·83	11·65
11·61	11·43
Mean 11·62	11·63

(6) Between B and D.

Equat. Diam.	Polar Diam.
11·67	11·87
12·13	11·53
11·67	11·33
11·46	
Mean 11·73	11·58

Mean of Equatoreal measures 11·704.
„ Polar „ 11·707.
General mean 11·705 ± 0″·030.

The equatoreal diameter, so called above, is merely the diameter parallel to the celestial equator, and the polar that at right angles to it.

If we reduce the result to distance unity, we get 6″·524, which corresponds to a diameter of 2,290 miles, assuming the solar parallax at 8″·85.

Mr. Libbey and Professor Rockwood also made a few observations: from the measurements of the former we get 11″·56, from those of the latter 11″·76. On the whole, I have thought it best, however, not to incorporate them with my own.

Princeton, N.J.,
1878, *November* 12.

Note on Some Remarks of Mr. Maxwell Hall on the Opposition of Mars. By David Gill, Esq.

In the last Number of the *Monthly Notices** Mr. Maxwell Hall has objected to the method of observing *Mars* with the heliometer as inferior in accuracy to the method of transits employed by him.

* This paper was communicated in November, along with Mr. Gill's papers which appeared in the last two Numbers of the *Monthly Notices*.—Ed.

I do not object to this criticism if Mr. Maxwell Hall can show that his results have smaller probable error and greater freedom from systematic error than mine. When his results are published it will be time enough to examine this question. I concede at once the advantages which the method of Mr. Hall affords in simplicity of reduction; but it appears to me that, when an exceptionally favourable opportunity, like the late opposition of *Mars*, occurs, the question is not what is the *easiest*, but what is the *best* method to observe it.

I object very strongly, however, to the proof which Mr. Maxwell Hall puts forward as to the accuracy of his observations. He contends that his observations are so accurate that by their means he has succeeded in determining the variation of the rate of his clock in short intervals, and makes the rather extraordinary statement that, though the rate of his clock may be relied upon from day to day for uniformity of rate to half a second, yet it varies two or three tenths of a second in intervals of ten or fifteen minutes. I am very much puzzled to find how Mr. Hall arrives at this conclusion and how he distinguishes between errors of observation and errors of clock rate.

The only proof of the supposed errors of his clock rate he finds in the various different values of the difference of R.A. between the same comparison stars obtained from observations at different times. It is obvious that these differences may be caused either by error in clock rate or by errors of observation; and, from the observations only, it seems to me impossible to say definitely to which of the two causes the discordances must be attributed. But, from what I know of clocks and their rates, the probabilities appear to me exceedingly strong that Mr. Hall himself is more likely to have made the errors in question than his clock. With regard to Mr. Hall's remarks which appear to imply that undue time has been occupied in my reductions, I think that the papers I have laid before the Society to-night are their best answer.

On the Observed Errors of Bouvard's Tables of Saturn.

By E. Dunkin, F.R.S.

During the discussion at the December meeting, after the reading of the Astronomer Royal's paper on the approaching conjunction of *Mars* and *Saturn*, reference was made to the large errors of Bouvard's Tables, on which the tabular places of *Saturn* given in the *Nautical Almanac* for 1879 depend. On that occasion I made a remark that the tabular errors of *Saturn*, deduced at Greenwich from recent observations, are not really so large as indicated by a direct comparison of the corresponding places inserted in the *Nautical Almanac* for 1879 and 1880, derived respectively from the Tables of Bouvard and Le Verrier. It

appears to me that the subject is sufficiently interesting to make it desirable to place on record in the *Monthly Notices* the actual amount of the observed tabular errors of *Saturn* during the last few years, determined from the Greenwich observations. On an examination of these errors, it will be perceived that their magnitude has been increasing of late, and the numbers in the following Table point to a further increase in the present year, though we can hardly anticipate that the amount of error will reach that found from the single comparison between the Tables of Bouvard and Le Verrier.

The numbers given below have been deduced from the Greenwich observations made near opposition, and when *Saturn* passed the meridian near 6^h, or near the time of the planet's quadrature with the Sun. From the observed errors of geocentric longitude and ecliptic polar distance of *Saturn* for the last eleven years, beginning with 1868, I have calculated the angular distances between the observed places and the tabular places, from which the general magnitude of the error from year to year can be easily seen. These, with the observed errors in R.A. and N.P.D., are given in the Table.

Year.	From Mean of Group nearest Opposition. Error in R.A.	Error in N.P.D.	Difference between Observed and Tabular Places.	From Mean of Group nearest 6^h M.T. Error in R.A.	Error in N.P.D.	Difference between Observed and Tabular Places.
	s	"	"	s	"	"
1868	−0·50	+5·02	8·8	−0·60	+ 3·51	9·1
1869	−0·86	+2·81	12·4	−0·94	+ 0·87	13·4
1870	−0·89	+1·65	12·5	−0·83	+ 0·44	12·3
1871	−0·86	−0·31	12·0	−0·56	− 1·07	7·8
1872	−0·71	−1·49	10·0	−0·34	− 2·72	5·5
1873	−0·75	−3·52	11·1	−0·25	− 4·92	6·1
1874	−0·60	−4·12	9·6	−0·16	− 6·50	6·9
1875	−0·78	−6·72	13·2	−0·25	− 8·30	9·1
1876	−0·83	−5·65	13·5	−0·18	− 9·24	9·7
1877	−0·62	−8·92	12·8	−0·27	−10·43	11·2
1878	−0·77	−8·03	14·1	−0·52	− 8·60	11·5
Corresponding numbers deduced from the *Nautical Almanac*, 1879-80.						
1879				−1·44	− 3·00	21·9

Assuming Le Verrier's Tables of *Saturn* to be correct, the tabular error of Bouvard's Tables in geocentric longitude on 1880, January 1, is $-18''\cdot72$, and that of ecliptic north polar distance is $-11''\cdot26$, equivalent to a difference of $21''\cdot9$ between the separate places as derived from the old and new Tables. On this day *Saturn* passes the meridian of Greenwich at $5^h\ 55^m$

M.T., and this difference therefore becomes comparable with the numbers in the last column deduced from observation.

Though the tabular error of Bouvard's Tables appears to be increasing at the present time, we should hardly expect to find so great an increase as 10″ in the interval between December 1878 and December 1879, viewed in conjunction with the column of errors determined from observations made near 6^h; and may we not therefore assume that the large difference found between the two calculated places for 1880, January 1, is partly due to small outstanding errors in the new Tables?

Blackheath,
1879, *January* 4.

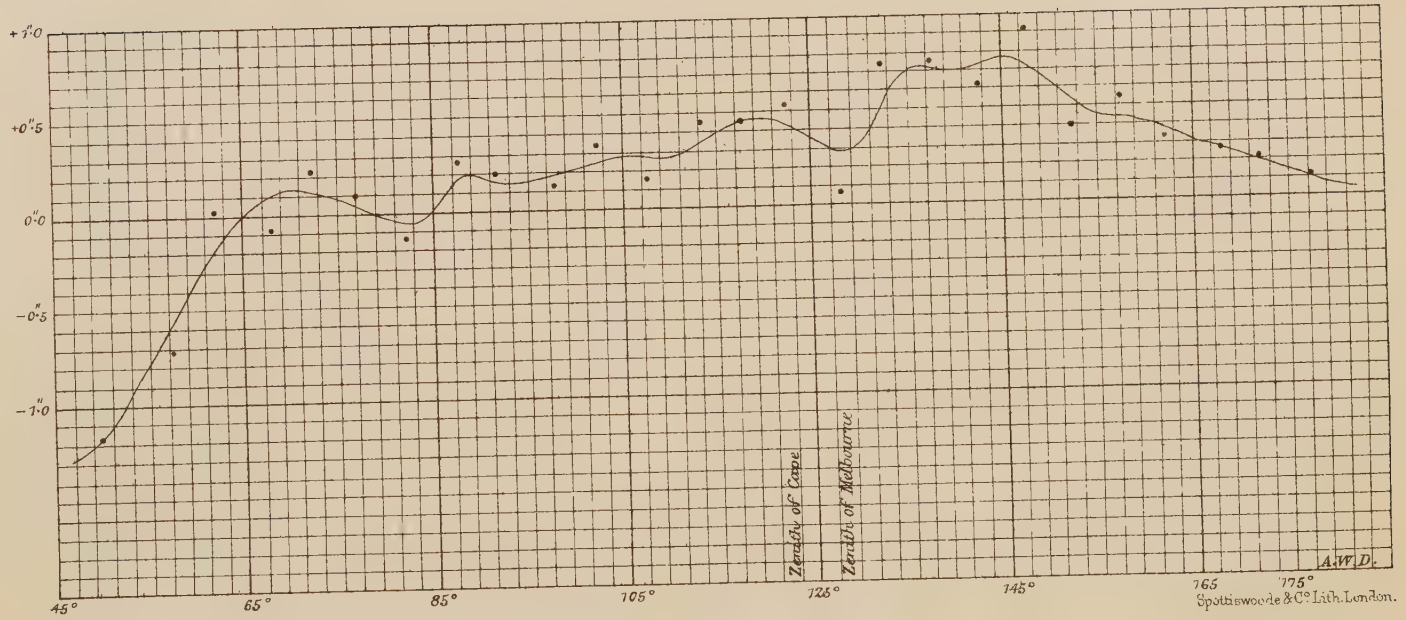
On the Reduction of the North Polar Distances of the First Melbourne General Catalogue for 1870 *to Auwers' Standard.*

By A. W. Downing, B.A. (Dublin).

This investigation consists essentially of a comparison of the North Polar Distances of the Cape Catalogue for 1860 with those of the Melbourne Catalogue. The reduction of the latter to the Standard is then accomplished by applying to the differences Cape—Melbourne for different N.P.D.'s the corrections applicable to the N.P.D.'s of the Cape Catalogue to reduce them to the Standard, which I have given in a former paper, printed in the *Monthly Notices* for December 1878.

There are 352 stars common to the Cape and Melbourne Catalogues which are available for the comparison, after rejecting those whose places in either Catalogue depend on a single observation. I have also rejected *α Centauri* and another star which gave a discordant result. As the southern stars have generally been reduced to the mean epochs of the Catalogues with different proper motions in each Catalogue, it has been necessary to correct the places given in one of the Catalogues for the difference, and the proper motion given in the Cape Catalogue has been adopted as the one to be used, except in those cases in which this proper motion is taken from the B.A.C., when the value given in the Melbourne Catalogue has been used in preference. The places in the Cape Catalogue have then been brought up to 1870, with the precessions given in this Catalogue and the adopted proper motion, and the difference taken between the N.P.D. thus obtained and that given in the Melbourne Catalogue, corrected, if necessary, for assumed proper motion. The 352 stars, having been arranged in order of N.P.D., have been taken in groups, each group embracing about 5°, and the means taken of the N.P.D.'s and of the differences between the Catalogues for each group. These mean differences have been

COMPARISON OF THE NORTH POLAR DISTANCES, OF THE CAPE
AND
MELBOURNE CATALOGUES.



laid down and a curve (*see* diagram) drawn through the points, which may be taken as representing the systematic differences between the Catalogues.

The following Table gives the differences as computed and as read off from the curve:—

N.P.D.		Number of Stars.	C−M.	
			Computed.	Curve.
°	′		″	″
49	51	4	−1·17	−1·17
57	13	3	−0·71	−0·59
62	10	10	+0·03	−0·20
67	56	9	−0·08	+0·09
72	14	7	+0·22	+0·12
77	5	12	+0·10	+0·04
82	23	14	−0·12	−0·06
87	20	11	+0·28	+0·16
91	47	10	+0·20	+0·16
97	52	8	+0·12	+0·18
102	39	5	+0·34	+0·25
107	22	9	+0·16	+0·26
113	3	7	+0·46	+0·34
117	37	20	+0·45	+0·45
122	24	12	+0·53	+0·44
127	45	20	+0·07	+0·28
132	25	30	+0·74	+0·53
137	18	25	+0·77	+0·71
142	46	14	+0·62	+0·73
147	36	20	+0·92	+0·72
152	27	21	+0·40	+0·52
157	19	28	+0·53	+0·44
162	19	6	+0·32	+0·37
168	9	14	+0·27	+0·27
172	15	20	+0·21	+0·19
177	13	13	+0·12	+0·11

The differences have then been read off from the curve for every 4°, and the correction to the Cape N.P.D. to reduce it to the Standard (or to Henderson, for stars south of N.P.D. 120°) applied. The result is given in the two following Tables:—

Table I.—*Reduction to Auwers' Standard.*

N.P.D. °	C−M. ″	Standard−M. ″	N.P.D. °	C−M. ″	Standard−M. ″
48	−1·25	−0·87	88	+0·17	+0·29
52	−1·05	−0·78	92	+0·14	+0·33
56	−0·70	−0·55	96	+0·17	+0·42
60	−0·35	−0·22	100	+0·22	+0·44
64	−0·05	+0·03	104	+0·27	+0·35
68	+0·10	+0·12	108	+0·26	+0·20
72	+0·12	+0·10	112	+0·30	+0·09
76	+0·06	−0·02	116	+0·43	+0·27
80	−0·03	−0·09	120	+0·46	+0·21
84	−0·04	−0·02	124	+0·37	+1·97

Table II.—*Reduction to Henderson.*

N.P.D. °	C−M. ″	H−M. ″	N.P.D. °	C−M. ″	H−M. ″
120	+0·46	+1·70	152	+0·55	+0·11
124	+0·37	+1·54	156	+0·44	−0·02
128	+0·28	+1·24	160	+0·41	+0·09
132	+0·47	+1·00	164	+0·34	−0·02
136	+0·72	+0·83	168	+0·27	−0·20
140	+0·70	+0·55	172	+0·19	−0·24
144	+0·76	+0·71	176	+0·12	−0·18
148	+0·69	+0·51	180	+0·06	−0·02

This reduction to Henderson is of course provisional only. I hope that, when Mr. Stone's great Catalogue is published, it will be possible to deduce a system of Standard North Polar Distances for Southern Stars analogous to that given by Dr. Auwers for stars visible in European latitudes.

Greenwich,
1879, *Jan.* 7.

Phenomena of Jupiter's Satellites, 1877–78, *observed at Stonyhurst Observatory*. By the Rev. S. J. Perry.

1877	Sat.	Phenomenon.	G.M.T. h m s	Remarks.
July 24	II	Occ. D. last contact	11 11 34·2	Poor; definition very bad; tremulous.
1878				
June 26	IV	Ec. R. first contact	14 50 14·0	Passing clouds; strong daylight.
27	III	Tr. egress, bisection	13 29 34·6	
		last contact	33 24·1	
29	III	Occ. D. first contact	12 35 30·5	Very unsteady.
		bisection	39 29·5	
		last contact	42 45·7	
Aug. 10	I	Tr. E. first contact	10 43 46·6	
		bisection	47 29·6	
		last contact	49 46·1	
25	I	Occ. D. first contact	9 14 4·1	
		bisection	15 47·0	
		last contact	17 5·5	
	I	Ec. R. first seen	12 19 31·2	
Sept. 9	I	Occ. D. first contact	9 56 54·2	Cloudy; definition very bad.
		bisection	10 0 0·7	
		last contact	2 51·7	
10	I	Occ. D. first contact	7 14 6·9	
		bisection	7 16 51·4	
		last contact	7 18 47·2	
	III	Occ. D. bisection	9 3 13·9	
		last contact	9 6 18·4	
	I	Ec. R.	10 38 42·4	Hazy.
20	II	Occ. D. first contact	8 53 26·9	
		bisection	8 55 42·4	
		last contact	8 58 19·4	Seen within the limb for about 45ˢ after this.
Nov. 4	I	Ec. R.	7 32 17·4	

The above observations were all made by Mr. W. Carlisle; and the shortness of the list is due mainly to the absence of the object-glass, which has been very successfully repolished by Mr. J. Simms, who felt confident that the spherical aberration could be fairly removed in the process.

Observations of Occultations of Stars by the Moon, and of Phenomena of Jupiter's Satellites, made at the Royal Observatory, Greenwich, in the Year 1878.

(*Communicated by the Astronomer Royal.*)

Occultations of Stars by the Moon.

Day of Obs. 1878.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Mar. 16	Disapp. of A Leonis	Altaz.	100	Dark	9 57 16·5	T
June 5	,, π^2 Cancri	E. Eq.	140	,,	8 54 16·6	,,
Sept. 6	,, h^1 Sagittarii	,,	,,	,,	7 32 27·8	HP
Nov. 10 (*a*)	,, 17 Tauri	S.E. Eq.	285	Bright	9 31 19·5	WC
,, (*b*)	,, 20 Tauri	,,	,,	,,	10 23 17·0	,,
,, (*c*)	Reapp. 17 Tauri	,,	,,	Dark	10 46 40·9	,,
,,	Disapp. η Tauri	E. Eq.	140	Bright	11 11 16·0	GP

Notes.

(*a*) The star disappeared gradually in a sort of luminous haze surrounding the Moon's limb, which seemed to retire from the star for a space of three or four seconds of time. Just before disappearing the star was seen apparently bisected by the limb.

(*b*) The star disappeared gradually at the Moon's bright limb; observed with a graduated dark shade.

(*c*) Reappeared instantaneously.

Phenomena of Jupiter's Satellites.

Day of Obs. 1878.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time from *N.A.* h m s	Observer.
July 4	III	Tr. ing. first contact	E. Eq.	140	13 8 26·2	13 12	C
17	I	Occ. reapp. first contact	,,	,,	13 58 45·8		
,,	I	,, bisection	,,	,,	14 0 15·5	14 0	AD
,,	I	,, last contact	,,	,,	14 2 0·2		
18	I	Tr. egr. last contact	,,	,,	11 7 18·4	11 10	T
,,	II	Ecl. disapp.	,,	,,	11 58 50·0	11 59 43	,,
29	III	Occ. disapp. first contact	,,	,,	12 40 49·5	12 41	C
Aug. 17 (*d*)	I	Tr. ing. first contact	S.E. Eq.	220	10 11 23·7		
,,	I	,, bisection	,,	,,	10 14 5·2	10 12	M
,,	I	,, last contact	,,	,,	10 16 24·8		
,, (*e*)	I	Tr. egr. first contact	,,	,,	12 28 24·2		
,,	I	,, bisection	,,	,,	12 30 58·8	12 32	,,
,,	I	,, last contact	,,	,,	12 33 33·3		

Day of Obs. 1878.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time from *N.A.* h m s	Observer.
Aug. 19 (*f*)	II	Occ. disapp. first cont.	E. Eq.	140	10 16 59·5		
,,	II	,, bisection	,,	,,	10 19 4·1	10 21	R
,,	II	,, last cont.	,,	,,	10 21 13·8		
26 (*g*)	I	Tr. egr. first contact	,,	,,	8 40 29·1		
,,	I	,, last contact	,,	,,	8 43 19·7	8 44	C
27 (*h*)	III	Ecl. reapp. first seen	S.E. Eq.	220	8 43 2·5		
,,	III	,, dichotomised	,,	,,	8 44 52·2	8 45 54	M
,,	III	,, full brightness	,,	,,	8 46 51·9		
Sept. 3 (*i*)	I	,, first seen	,,	,,	8 43 18·4		
,,	I	,, full brightness	,,	,,	8 44 39·2	8 43 16	WC
,, (*k*)	III	Occ. reapp. first appearance	,,	,,	9 1 43·4		
,,	III	,, bisection	,,	,,	9 3 26·1	9 8	,,
,,	III	,, last contact	,,	,,	9 4 55·8		
,,	III	Ecl. disapp. first obs.	,,	,,	9 12 54·5		
,, (*l*)	III	,, disappearance	,,	,,	9 16 38·9	9 18 4	,,
6 (*m*)	II	Ecl. reapp. first seen	E. Eq.	140	8 58 5·0		
,,	II	,, half brightness	,,	,,	8 59 12·8	8 58 39	HP
,,	II	,, full brightness	,,	,,	9 0 2·7		
18 (*n*)	I	Tr. egr. first contact	S.E. Eq.	220	8 32 46·2		
,,	I	,, bisection	,,	,,	8 36 0·6	8 36	M
,,	I	,, last contact	,,	,,	8 39 5·1		
,, (*o*)	I	,, last contact	E. Eq.	140	8 41 11·7	8 36	W
,,	IV	Ecl. reapp. first seen	S.E. Eq.	220	9 25 9·3		
,, (*p*)	IV	,, dichotomised	,,	,,	9 28 47·0	9 34 49	M
,,	IV	,, full brightness	,,	,,	9 32 56·3		
,, (*q*)	IV	,, first seen	E. Eq.	140	9 27 59·0		
,,	IV	,, dichotomised	,,	,,	9 29 33·7	9 34 49	W
,, (*r*)	IV	,, first seen	Altaz.	100	9 29 38·1	9 34 49	C
19	I	,, first seen	,,	,,	7 3 3·9	7 2 45	R
20	II	Occ. disapp. first cont.	E. Eq.	140	8 57 3·8		
,,	II	,, last contact	,,	,,	9 0 18·2	8 58	C
26 (*s*)	IV	Tr. egr. last contact	S.E. Eq.	220	8 6 0·5	8 25	WC
,, (*t*)	I	Ecl. reapp. first seen	,,	,,	8 58 3·9		
,,	I	,, half brightness	,,	,,	8 58 51·8	8 58 7	,,
,,	I	,, full brightness	,,	,,	9 0 1·6		
,,	I	,, first seen	E. Eq.	140	8 58 14·8		
,,	I	,, half brightness	,,	,,	8 59 17·7	8 58 7	J
,,	I	,, full brightness	,,	,,	9 0 27·5		

Day of Obs. 1878.	Satellite.	Phenomenon.	Telescope.	Power.	Mean Solar Time of Observation. h m s	Mean Solar Time from *N.A.* h m s	Observer
Oct. 11 (*u*)	I	Tr. ing. first contact	S.E. Eq.	285	6 22 2·3		
,,	I	,, bisection	,,	,,	6 24 31·9	6 20	M
,,	I	,, last contact	,,	130	6 26 41·6		
18	I	Tr. ing. first contact	E. Eq.	140	8 11 41·6		
,,	I	,, bisection	,,	,,	8 14 41·1	8 15	J
,,	I	,, last contact	,,	,,	8 18 20·5		
23 (*v*)	III	Occ. disapp. bisection	S.E. Eq.	220	7 52 12·0	7 58	WC
24 (*w*)	II	Tr. egr. last contact	E. Eq.	140	6 34 41·5	6 28	AD
Nov. 2	II	Ecl. reapp. first seen	,,	70	5 56 0·6		
,,	II	,, full brightness	,,	140	5 58 24·2	5 56 25	,,
11 (*x*)	I	Occ. disapp. first contact	,,	,,	5 54 54·2		
,,	I	,, bisection	,,	,,	5 55 44·1	5 55	,,
,,	I	,, last contact	,,	,,	5 56 48·9		

Notes.

(*d*) *Jupiter* was occasionally very well defined, especially at the first contact. The limb of the planet became afterwards very boiling and rugged. The satellite was very brilliant on the disk.

(*e*) Limb tremulous. The satellite did not appear so bright as at ingress.

(*f*) The planet well defined.

(*g*) The image of the planet very bad.

(*h*) *Jupiter* and the satellites were very tremulous. The time noted at "first seen" may be two or three seconds late.

(*i*) Pretty exact; a very minute speck when first seen; the Airy eye-piece used throughout.

(*k*) The time noted at first appearance is pretty exact.

(*l*) A very faint speck at time of disappearance.

(*m*) Satisfactory.

(*n*) Very tremulous, especially at the last contact.

(*o*) The image very diffused; the observation difficult.

(*p*) The times recorded at "dichotomised" and at "full brightness" were a little early; probably they should be increased by 20ˢ. Images very tremulous.

(*q*) Rather late; the time recorded at full brightness is also uncertain, owing to tremor and diffusion.

(*r*) Not certain; thin clouds were continually passing.

(*s*) The satellite was not seen a few minutes before the last contact when on the disk. Definition very bad. The Airy eye-piece used.

(*t*) Increased very rapidly in brightness in the next two or three seconds.

(*u*) Very tremulous; definition bad.

(*v*) Not satisfactory; the planet was only seen for about a minute, the sky being cloudy at the first and last contacts. The Airy eye-piece used.

(*w*) The first phases could not be observed, owing to the bad image.

(*x*) Definition good.

The clear aperture of the object-glass of the S.E. Equatoreal is 12¾ inches, of the East Equatoreal 6·7 inches, and of the Altazimuth 3¾ inches.

The initials WC, C, AD, M, T, W, HP, R, GP, and J, are those of Mr., Christie, Mr. Criswick, Mr. Downing, Mr. Maunder, Mr. Thackeray, Mr. Wickham, Mr. Pead, Mr. Robinson, Mr. Pearce, and Mr. James.

Royal Observatory, Greenwich,
1878, *December* 31.

Ephemerides for Determining the Positions of the Satellites of Uranus, 1879.

By A. Marth, Esq.

Angles of position, p, of the major axes and logarithms of the major and minor semi-axes, a and b, of the apparent orbits of the satellites.

Greenwich. Noon. 1879.		Ariel.		Umbriel.		Titania.		Oberon.	
	p_o	log a	log b	log a	log b	log a	log b	log a	log b
	°								
Jan. 25	13·03	1·1808	0·5110	1·3248	0·6550	1·5397	0·8699	1·6660	0·9961
30	12·99	·1818	·5185	·3258	·6625	·5407	·8774	·6669	1·0036
Feb. 4	12·95	·1825	·5260	·3265	·6699	·5414	·8848	·6677	·0111
9	12·90	·1831	·5334	·3271	·6774	·5420	·8923	·6682	·0185
14	12·85	·1835	·5407	·3275	·6847	·5424	·8996	·6686	·0258
19	12·81	·1837	·5479	·3277	·6918	·5426	·9068	·6688	·0330
24	12·76	·1837	·5547	·3277	·6967	·5426	·9136	·6688	·0399
Mar. 1	12·71	1·1835	0·5613	1·3275	0·7052	1·5424	0·9201	1·6686	1·0464
6	12·66	·1831	·5674	·3271	·7113	·5420	·9262	·6683	·0525
11	12·61	·1826	·5730	·3265	·7170	·5414	·9319	·6677	·0581
16	12·57	·1818	·5781	·3258	·7221	·5407	·9370	·6669	·0631
21	12·52	·1809	·5826	·3249	·7266	·5398	·9415	·6660	·0677
26	12·48	·1798	·5866	·3238	·7305	·5387	·9455	·6649	·0717
31	12·44	·1786	·5899	·3225	·7339	·5375	·9488	·6637	·0750
Apr. 5	12·41	1·1772	0·5925	1·3212	0·7365	1·5361	0·9514	1·6623	1·0777
10	12·38	·1757	·5945	·3197	·7385	·5346	·9534	·6608	·0797
15	12·36	·1741	·5959	·3180	·7399	·5330	·9548	·6592	·0810
20	12·34	·1724	·5966	·3163	·7405	·5312	·9554	·6575	·0817
25	12·32	·1706	·5966	·3145	·7405	·5294	·9555	·6557	·0817
30	12·31	·1687	·5959	·3127	·7399	·5276	·9548	·6538	·0810
May 5	12·30	1·1668	0·5946	1·3107	0·7386	1·5257	0·9535	1·6519	1·0798
10	12·30	·1648	·5927	·3088	·7367	·5237	·9516	·6499	·0778
15	12·31	·1628	·5902	·3068	·7341	·5217	·9490	·6480	·0753
20	12·32	·1608	·5870	·3048	·7309	·5197	·9459	·6460	·0721
25	12·34	1·1589	0·5832	1·3028	0·7271	1·5178	0·9420	1·6440	1·0683

Longitudes of the satellites in their orbits reckoned from the points where they are at their greatest northern elongations.

Greenwich, Noon.	Ariel. long.	diff.	Umbriel. long.	diff.	Titania. long.	diff.	Oberon. long.	diff.
	°	°	°	°	°	°	°	°
Jan. 25	39·65		26·09		327·57		329·31	
		714·18		434·33		206·74		133·67
30	33·83		100·42		174·31		102·98	
		·17		·33		·73		·67
Feb. 4	28·00		174·75		21·04		236·65	
		·16		·32		·73		·67
9	22·16		249·07		227·77		10·32	
		·16		·32		·72		·67
14	16·32		323·39		74·49		143·99	
		·15		·32		·73		·66
19	10·47		37·71		281·22		277·65	
		·14		·31		·72		·67
24	4·61		112·02		127·94		51·32	
		·14		·31		·72		·66
Mar. 1	358·75		186·33		334·66		184·98	
		·14		·30		·72		·66
6	352·89		260·63		181·38		318·64	
		·13		·30		·72		·67
11	347·02		334·93		28·10		92·31	
		·12		·30		·73		·66
16	341·14		49·23		234·83		225·97	
		·12		·30		·72		·66
21	335·26		123·53		81·55		359·63	
		·12		·30		·72		·67
26	329·38		197·83		288·27		133·30	
		·11		·30		·72		·67
31	323·49		272·13		134·99		266·97	
		·11		·30		·72		·67
Apr. 5	317·60		346·43		341·71		40·64	
		·11		·29		·73		·67
10	311·71		60·72		188·44		174·31	
		·11		·30		·73		·67
15	305·82		135·02		35·17		307·98	
		·11		·30		·73		·68
20	299·93		209·32		241·90		81·66	
		·11		·30		·73		·68
25	294·04		283·62		88·63		215·34	
		·11		·30		·73		·68
30	288·15		357·92		295·36		349·02	
		·12		·31		·74		·69
May 5	282·27		72·23		142·10		122·71	
		·11		·30		·74		·68
10	276·38		146·53		348·84		256·39	
		·12		·31		·74		·69
15	270·50		220·84		195·58		30·08	
		·12		·31		·75		·70
20	264·62		295·15		42·33		163·78	
		714·12		434·32		206·75		133·70
25	258·74		9·47		249·08		297·48	

These values are to be interpolated for the times for which the positions of the satellites are required. The position angles, p, and distances, s, are then found by means of the equations—

$$s \sin (p_o - p) = b \sin \text{long.}$$
$$s \cos (p_o - p) = a \cos \text{long.}$$

Notes on the late Admiral Smyth's "Cycle of Celestial Objects," Volume the Second, commonly known as the "Bedford Catalogue."
By Herbert Sadler, Esq.

In the year 1844 Admiral (then Captain) Smyth published his *Cycle of Celestial Objects*, for the second volume of which, commonly known as the "Bedford Catalogue," the author in the ensuing year received the Gold Medal of the Society. I may be mistaken, but it seems to me that the Presidential Address on that occasion is couched in eminently cautious language as to the exactness of the micrometrical measures in Admiral Smyth's work. If this be the case, it has been abundantly justified by the results. The following facts may perhaps convey some idea of the intrinsic value of the measures recorded therein. In the first hour of right ascension there are 108 measures (54 of the position angles and 54 of the distances) of 36 different objects, being stars with comites, double, triple, or multiple stars. Of these 108 measures, no less than 32 have the mark "w^1" attached to them; a weight, as the author himself observes, representing nearly worthlessness. More than this, 12 out of these 36 objects have this weight assigned to the measures both of their position angles and distances, so that the measures of *one-third* of the objects in the first hour of R.A. are avowedly useless for any practical purpose, and there is a residuum of 8 more such measures to be distributed amongst the remaining 24 objects. On presenting the medal, the Astronomer Royal, observing that the character of the Council was most deeply pledged in the award, requested that Captain Smyth would present the original observations on which the Catalogue was based to the Society, in order that, if occasion should arise, these MSS. might be readily consulted by any Fellow. In presenting these, Captain Smyth wrote as follows: "Previously, however, to its removal [from Hartwell], a most careful scrutiny has been instituted of the slips and other papers with the printed *Cycle*, and various typographical errors have been detected in consequence. Some printed lists of these are herewith forwarded for any Fellows of the Society who may possess a copy of the work." As possibly every copy of the *Cycle* does not possess this list, I have transcribed the portion relating to vol. ii., and inserted it at the end of these Notes. Some few years ago, being much interested at the time in certain stars of the "Bedford Catalogue" which had apparently disappeared, I took the earliest opportunity I had of examining these original MSS., and was astounded to find the most extraordinary discrepancies between the printed Catalogue and the originals—discrepancies which at once accounted for the apparent anomalies. About the same time Mr. Burnham was sending to the *English Mechanic* some most admirable and searching criticisms on

Smyth's measures. As the opinion of so eminent an observer will carry deserved weight with it, I may be permitted to quote the following words from one of his letters on the subject: "No publication of original observations, in this or in any other language, can be named which contains so many serious errors. The measures of the Struves, Dembowski, Dawes, Secchi, and half a dozen others whose names might be mentioned, do not contain altogether more than a small fraction of the mistakes in the *Cycle* which have led to so much discussion and confusion. . . Ordinarily there is no difficulty in detecting the mistake at once. This is not the case with the *Cycle*. There is no theory which will account for the many serious discrepancies. The measures generally agree substantially with those which are given from prior observers, but the strangest part is that this agreement is kept up just the same where the earlier measures were all wrong." As far as I am aware, there is one Catalogue only, and that not an original one, which surpasses the "Bedford Catalogue" in inaccuracy, and that Catalogue is the "Reference Catalogue of Multiple and Double Stars," forming vol. xl. of the *Memoirs*. I have not included in my Notes any notices of errors in Smyth's measures of three or four seconds of arc in the distances, or two or three degrees in the position angles, such as those that occur in the measures of the companions of ζ *Persei*, 72 *Virginis*, 15 *Hydræ*, γ *Aquarii*, and many other stars; nor have I alluded to mistakes in the magnitudes, alignments, diagrams, or descriptions, as such a course would have swelled these notes to an inconvenient length, and, indeed, would have been almost superfluous, as the proven inaccuracy of so many measures throws the greater doubt on all the others. Sir George Airy, in his address on the occasion of presenting the Gold Medal to Admiral Smyth, observed: "When the question shall be put regarding the measures of the 'Bedford Catalogue,' made at a critical time, and on which a future theory may hinge—Can these numbers be trusted with certainty to one or two-tenths of a second?—shall we be able to answer—Without doubt they can." Of this the Council were apparently satisfied, or else they would presumably not have awarded the medal to Admiral Smyth. I have thought it better, therefore, as the charge I have brought against the Bedford Catalogue is of a very serious character, to place an asterisk against the symbol of the observer whose erroneous measure Smyth appears to have followed, so that anyone may be able to detect the source of Smyth's error at a glance in cases where he has presumably copied the measures of others; but, for the sake of brevity, I have only given one or two correct measures for comparison. The symbols used are those commonly employed.

1878, *Dec.* 13.

α *Cassiopeiæ*. *Cycle*, No. 20.

	°	″	
H.	275·4	56·2	1781·97
*H_2.	278·8	90· ±	IV. Cat.
Sm.	278·4	96·9	1831·86
Ja.	279·5	61·4	1856·7

Sm. says (*Cycle*, p. 12): "The difference in distance is so remarkable that it must be imputed to instrumental error [on the part of H] rather than that the acolyte is describing an ellipse round its primary." He re-examined it in 1851, on a friend's pointing out the mistake to him, and deduced a result closely agreeing with Jacob's. (*Spec. Hart.*, p. 217.)

P. I. 222. *Arietis*. *Cycle*, No. 78.

	A–B °	A–B ″	A–C °	A–C ″	A–D	A–D		A m	B m	C m	D m
Σ.	53·5	2·4	167·4	39·5	caret	caret	1832·4	8·5	11·0	9·2	caret.
Sm.	53·0	2·5	165·0	40·0	359·2	165·0	1834·9	6	15	10	9
D.	53·7	2¼	165·6	36·2	1·6	182·5	1862·9	9	10½	9¼	6¼

(*Cf. Monthly Notices*, vol. xxiii., pp. 11, 78, 93.) Smyth's diagram agrees exactly with his description of this object in the Bedford Catalogue. Kn. gives 183″·7 for distance of D, (1862·9).

55 *Cassiopeiæ*. *Cycle*, No. 84.

Hind gives $0^h\ 18^m\ 2^s{\cdot}1$ +63° 28′ 17″ (1878·0) for the place of *Nova* 1572, after D'Arrest.

θ *Persei*. *Cycle*, No. 109.

	A–C °	A–C ″	
Sm.	219·0	27·0	1833·6
Flam.	218·5	68·0	1877·6

Smyth's diagram agrees with his published results. His distance is totally erroneous, and is probably owing to H's remark that there is a third star *within* 1″.

γ *Persei*. *Cycle*, No. 124.

	°	″	
*H_2.	224·9	60	Vth Cat.
Sm.	226·0	55·0	1837·6
Bu.	324·1	caret	1876

Typographical error in Herschel's Vth Catalogue of 224°·9 for 324°·9, which Smyth has followed.

β Persei. Cycle, No. 127.

	°	″	
Sm.	195·0	55·0	1835·6
Bu.	192·4	81·9	1878·6

I do not know of any other observations of this star. Smyth's distance is evidently erroneous.

19 *Pleiadum. Cycle*, No. 138.

	°	″	
Sm.	335·0	45·0	1835·01
Main.	331·1	64·6	1863·1

I have not seen any other measures.

τ Orionis. Cycle, No. 196.

	A–B		A–C		
	°	″	°	″	
*H_2	250·4	18·0	63·8	18·0	Vth Cat.
Sm.	255·0	15·0	65·0	20·0	1835·9
Bu.	250·1	35·98	59·8	35·97	1878·2

Burnham has found B to be a close double. Smyth has followed Herschel II.

ν Aurigæ. Cycle, No. 229.

	°	″	
H_2.	208·2	53·72	1782·7
Sm.	201·9	85·0	1833·7
Bu.	207·0	55·0	1877·8

331°·80 in *Hh.* (p. 82) is a mistake. Sm. remarks: "The discordance in the distance is very great."

α Orionis. Cycle, No. 231.

	°	″	
*H_2.	152·3	161·8	1780·9
Sm.	155·0	160 0	1832·7
Bu.	152·3	174·7	1877·9

θ Aurigæ. Cycle, No. 233.

	°	″	
*H_2.	286·0	35·3	1782·7
Sm.	289·0	30·0	1832·6
OΣ.	290·5	48·3	1852·1
Bu.	292·7	45·5	1878·0

OΣ has lately discovered A to be a close double (5°·5 : 2″·15, '71). There are several other comites.

β Canis Majoris. Cycle, No. 246.

	°	"	
Sm.	339·0	104·0	1833·7
Bu.	339·7	183·9	1877

Smyth's distance is preposterous.

61 *Geminorum. Cycle*, No. 286.

	A–B °	"		A m	B m
Sm.	110·0	60·0	1835·8	7½	9

Mr. Webb mentions (*Monthly Notices*, xxxv., p. 340; *Celestial Objects* (3rd edition), p. 247) that on two occasions, in 1852 and 1855, he failed to see Smyth's 9ᵐ star. On the latter occasion, however, he found an exceedingly minute star, at about the right distance, with an estimated position angle of 185° or 190°. Mr. Knott also failed to see any star in the position of the one given by Smyth in the Bedford Catalogue, but noticed a minute star having a position angle of 170° or 175° at a distance of 75″. This star is the same as the one mentioned by Mr. Webb. I find, on referring to Smyth's diagram of this object in the MSS. of the Bedford Catalogue, that the position angle is undoubtedly about 170°. As he only credited his measures with a weight of 1—a weight which, by his own admission, represents nearly worthlessness—the apparent alteration in the distance may be easily explained. Smyth's magnitudes are so exceedingly vague and inaccurate that no suspicion of variability can be entertained for a moment. Sm. gives the distance of the companion to the next star but one in the *Cycle* (63 *Geminorum*; *Cycle*, No. 288) as 50″. H, in 1783, gave 44″·25; Main, in 1863, 44″·61. Mistakes of this kind, however, occur so frequently in the Bedford Catalogue as to be hardly worth noticing in detail.

β Canis Minoris. Cycle, No. 289.

	A–B °	"	A–C °	"	
Sm.	80·0	35·0	312·0	105·0	1831·8
Bu.	73·2	120±	311·2	141·8	1877·1

Burnham observes of the discrepancies between the published distances of these companions and of the comes of *β Canis Majoris* and his own measures: "Of course any such change since Smyth's observations is out of the question."

45 H IV. *Geminorum. Cycle*, No. 290.

	°	′	
*H_2.	355		1833
Sm.	355·0	95·0	1836·2
OΣ.	2·38	100·12	1853·2
Kn.	2·44	100·16	1864·9

OΣ remarks (*Mélanges* etc., iii., p. 572): "La direction 85° *n.p.*, estimée par Sir John Herschel, est erronée. Probablement il faut lire *n.f.* au lieu de *n.p.* Cependant il est bien curieux que la même erreur soit commise par l'amiral Smyth dans son Bedford Catalogue."

α Canis Minoris. Cycle, No. 298.

	°	″	s	
Sm.	85·0	145·0 (MSS. Δ R.A. 18·3)	1833·81	B 8m.

This is the celebrated missing star near *Procyon*. A moment's inspection will show that the two distances given by Smyth are irreconcilable the one with the other. Taking the proper motion of *Procyon* into account, the estimate Δ R.A. = 18s·3 will accord fairly well with the actual distance of a small star of about the 9th magnitude, which has been found to be a close pair by Bird, and independently detected to be such by Burnham and Dembowski. The following are some of the measures:—

	°	″	″	
Se.	83·60	331·62 (misprinted 33·162)		1856·16
Morton.	83·89	327·6		1857·92
Powell.	83·1	332·2		1860·82
Flammarion.	80·5	346·5		1877·17

175 P. VII. *Argus. Cycle*, No. 301.

	°	″	
*Piazzi. (reduced)	325·0	10·5	1800·0
Sm.	326·8	9·8	1831·9

Smyth's position angle is quite erroneous. Ja. gives 318°·8 (1846·2).

δ Cancri. Cycle, No. 335.

	°	′	
*H_2.	160	25	II Cat.
Sm.	163·0	25·0	1838·2
Lamont.	123·8	27·94	1836·2
Bu.	113·9	40·97	1878·2

Cf. "*Observatory*," No. 14. Smyth has copied Herschel's fallacious measure.

78 H I. *Ursæ Majoris. Cycle*, No. 365.

Sm. R.A. 9h 34m 52s, 1840·0. D'Arrest, in his *Sid. Neb. Obs. Hav.*, points out that Herschel's R.A. of this nebula is 1m too little. Smyth, though he professes to have determined its place independently, has fallen into the same error. Taking D'Arrest's R.A. as correct, Smyth's should be 9h 35m 47s, 1840·0.

ψ *Leonis*. *Cycle*, No. 366.

ψ *Leonis* is not variable, as stated in the Bedford Catalogue. It is probably confused with R *Leonis*, the place of which for 1840·0 is $9^h\ 38^m\ 57^s + 12°\ 10'·0$.

γ *Leonis*. *Cycle*, No. 376.

M. Flammarion remarks (*Catalogue des Étoiles doubles et multiples etc.*, p. 59): "Il y a encore un autre point douteux. Sm. qui a mesuré tant de compagnons éloignés, n'a pas mesuré celui-ci [viz. C at 292°·8 : 229″·3 : 1877], quoiqu'il ait mesuré cinq fois γ de 1831 à 1843, et il a écrit: 'There are two stars in a line with A in the *n.p.* quadrant.' C'est évidemment l'une de ces étoiles. Il y en a une autre, D, de huitième grandeur, plus éloignée que C, et formant avec elle angle de 328° ± à une distance d'environ le tiers de AC, et une autre, E, de dixième grandeur plus loin encore, à un distance un peu plus grande que celle de CD, et un peu plus au nord: elles sont presque sur une même ligne avec C, mais D n'est pas du tout sur la même ligne que C relativement à A, et pour quelle s'y soit trouvée il y a 40 ans, il faut que le déplacement ait été plus grand que celui de γ." On referring to the diagram of this object in the MSS., I find that C and D are by no means in the same line with A. Many of the descriptions of such objects in the text of the Bedford Catalogue are deplorably inexact, and no reliance whatever can be placed in them.

β *Leonis*. *Cycle*, No. 425.

	°	″	
Sm.	114·0	298·0	1833·5

Smyth's position angle should be increased by 90°. It is not at all probable, as Burnham has remarked, that the small star seen by Kn. is Smyth's, and is therefore a variable.

62 H_8 IV. *Ursæ Majoris*. *Cycle*, No. 452.

	h	m	s	
Sm. RA.	11	47	3	1840·0

Herschel's R.A. is, according to D'Arrest, 1^m too small. Smyth states that "its mean apparent place was obtained by differentiation from that of γ *Ursæ Majoris*." It is remarkable, therefore, that his R.A. is also 1^m behind the correct one. It should be $11^h\ 48^m\ 6^s$, 1840·0.

53 *Virginis*. *Cycle*, No. 472.

	°	″		
$*H_2$.	30·0 ±	50	V. Cat.	"Position by diagram."
Sm.	35·0	45·0	1833·4	
Bu.	9·3	70·56	1878·2	

61 *Virginis. Cycle*, No. 477.

	°	′	
*I_2^H.	345·0	73·25	1783·0
Sm.	340·6	Δ R.A. 2ᵐ·8	1832·3
Kn.	22·6	169·29	1862·3
Bu.	25·2	189·34	1878·3

Smyth's observation is incomprehensible on the assumption that he really examined the star.

10 [54] *Hydræ. Cycle*, No. 519.

	°	″	
I_3^H.	128·2	11·29	1783·0
*H. & S.	136·7	9·95	1822·9
Sm.	138·4	9·8	1831·5
Howe.	129·8	9·68	1876·4

Smyth says that the measures of H. and S. show "a considerable direct motion in the elapsed time, which my measures appear to substantiate."

212 P. XIV. *Libræ. Cycle*, No. 524.

	A–B °	″			B–C °	″	
S. & H.	270·1	10·82	1823·3	*H_2	321·5	20·0	V. Cat.
Sm.	272·6	10·3	1833·4				
					A–C		
Ja.	284·0	13·34	1856·4	Sm.	320·0	20·0	1833·4
Bu.	290·8	15·30	1878·3	Bu.	322·2	120·6	1878·3

H_2 gives the position of B–C correctly. Smyth has overlooked the fact that Herschel measured B–C, and has given almost the same measures for A–C. There is no misprint in the Bedford Catalogue, as the diagram of the object in the MSS. agrees exactly with the *Cycle*. (*Cf. English Mechanic*, Nos. 687 and 690, May 24 and June 14, 1878.)

ι^1 *Libræ. Cycle*, No. 532.

I_3^H.	59·05	1781·4
*H. & S.	50·63	1822·8
Sm.	51·3	1837·4
Bu.	57·5	1878·4

Here Smyth has followed Herschel and South's erroneous measure.

764 H II. *Draconis. Cycle*, No. 550.

Sm. R.A. $15^h\ 35^m\ 53^s$, 1840·0. Using D'Arrest's correction, this should be $15^h\ 35^m\ 25^s$. The R.A. of Herschel II. is also in error.

39 *Serpentis. Cycle*, No. 554.

	°	″	
Sm.	355·0	12·0	1835·57

Herschel II. says of this object (*Memoirs*, vol. xl., p. 139): "This star, 39 *Serpentis*, is described by Smyth as identical with H III. 25; but it is H III. 25 [45 in original, but evidently 25] described by H as 39 *Serpentarii*, not *Serpentis*, and is accordingly set down in *h's* synopsis of H*h.* as 39 *Ophiuchi* (another name of *Serpentarius*) to avoid the equivoque of the abbreviation *Serp.*, and is in R.A. 17^h, N.P.D. 114°. It is remarkable, however, that the measures given by H and by Smyth agree both in position and distance, and both agree with those of 36 *Ophiuchi* (which see).—J. H." Smyth says of 39 *Serpentis*: "This object is 25 H III., and was thus registered by its discoverer. Pos. 357° 14′, Dist. 10″·0, Ep. 1780·65." Sm. gives for 36 *Ophiuchi* (*Cycle*, No. 607) 355°·6 : 11″·6 : 1830·63; 356°·2 : 12″·1 : 1838·52; and remarks: "It is 25 H III., . . . and was thus micrometrically measured at Slough—Pos. 357°14′, Dist. 10″·03, 1782·46." 39 *Serpentis* has no companion, and it is abundantly evident that Smyth mistook Herschel's *Serp.* (i.e. *Serpentarii*) for *Serpentis*, and has *measured*, therefore, a companion which has no existence, and moreover was careful that his measures should agree with the supposed prior observation of H. It is noteworthy that these *measures* are stated to have been made in the year 1835, in which year he did not make any measure of 39 *Ophiuchi*. Smyth actually gives the colour of the star he did not observe!

136 P. XVI. *Herculis. Cycle*, No. 580.

Sm. R.A. $16^h\ 30^m\ 26^s$, 1840·0. It should be $16^h\ 31^m\ 26^s$, 1840·0, on the authority of the *Catalogus Generalis*.

μ^1 *Sagittarii. Cycle*, No 639.

	A–B °	′	A–C °	″	A–D °	″	
*H_2.	262·5	12	313·8	45	115·2	40	V. Cat.
Sm.	260·0	10·0	315·0	40·0	114·5	45·0	1835·6
Bu. 1878·4	259·2	16·9	312·9	48·6	115·4	50·5	Ja. 1847·3

185 and 186 P. XIX. *Antinoi. Cycle*, Nos. 702 and 703.

Cf. Dawes (*Memoirs*, vol. xxxv., pp. 494–496; *Notices*, vol. xxiii., pp. 34–37); Hunt (*Notices*, vol. xxxii., pp. 90, 91, &c.).

Smyth's position angles and distances are utterly inaccurate; and, to make matters worse, he has followed the erroneous results of H. Full explanations of the matter will be found in the papers cited above.

54 *Sagittarii*. *Cycle*, No. 705.

	A–B °	A–B ″	A–C °	A–C ′	
*H_2.	13·6	25 ±	273·0	15· ±	IV. Cat. (No. 1424)
Sm.	42·8	28·5	280·0	20 0	1837·58
Main.	41·4	45·90	caret.		1861·73
Bu.	41·7	45·57	244·5	35·76	1878·7

Herschel's 13°·6 is probably a misprint for 43°·6. A–B are first mentioned in Herschel's 2nd Catalogue, as H. 599, and the measures therein recorded are 6, 10 : 40° : 40″ : 1826·58. Smyth did not notice this, and so copied the error in Herschel's measure in Catalogue IV. Ja. gives for the distance of A–B 45″·6 in 1845·8. Smyth refers in the text to Herschel's observation of C in the IVth Catalogue (No. 1424), and has followed Herschel's inaccurate observations of that star also. Considerable mistakes in the position angles and distances of small stars are not uncommon in the Catalogues of the younger Herschel.

295 P. XIX. *Cygni*. *Cycle* 719.

This is *not* Kirch's variable, which is between χ (Flamsteed) and P XIX. 295. Sm. has followed Baily and Piazzi. (*Cf.* Stone in *Monthly Notices*, vol. xxvi., p. 273.)

2 P. XX. *Aquilæ*. *Cycle*, No. 733.

Sm. R.A. 20ʰ 1ᵐ 51ˢ, Dec. + 16° 26′·8, 1840·0. Smyth's R.A. should be increased by 26ˢ, and his Decl. diminished by 7′·0.

o^2 *Cygni*. *Cycle* No. 739.

	A–D °	A–D ′	A–D ″			A–B °	A–B ″	
*Σ. (as quoted by Sm.)	333	41·9	337·38	1835·95	*H_2.	332·8	20	H. 1945
Sm.	333·8		338·0	1838·67	Sm.	330·0	15·0	1838·67
Σ (*M. M.*, p. 270)	323	41·9	337·833	1835·95	Bu.	321·7	36·82	1878

Here Smyth *quotes* Struve's position angle as 333° 41′·9, with which his own position angle, credited with a weight of 9, agrees within the *tenth* of a degree. Herschel's distance of A–B is largely in error, and Smyth, as is usual, has followed him. The Bedford MSS. give 333°·8 for A–D, so that there is no misprint.

178 P. XX. *Delphini. Cycle*, No. 751.

	A-C °	″	
*H_2.	121·5	20 0	IV. Cat.
Sm.	125·0	20·0	1835·91
Bu.	108·4	23·4	1878·2

β Delphini. Cycle, No. 756.

	A-C °	″	
*H_2.	107·7	18·0	V. Cat.
Sm.	105·5	15·0	1834·79
Lamont.	112·8	33·1	1836·8
Bu.	115·7	27·4	1878·6

Lamont gives 202° 49′·5, which I have diminished by 90°. Bu. has found A to be a close double.

α Cygni. Cycle, No. 760.

	°	″	
I_H^H.	90·0 ±	60·0 ±	1790·69
*H_2.	104·1	Δ R.A. 7ˢ·3	IV. Cat.
Sm.	102·5	108·5	1837·65
Bond I.	88·3	95·5	1848

β Equulei. Cycle, No. 784.

	A-B °	″	B-b °	″	A-C °	″	
*H_2.	314·4	40·0	14·5	2·0	278·0	50·0	V. Cat.
Sm.	317·0	35·0	15·0	3·0	275·0	50·0	1836 68
Bu.	308·7	67·4	10·4	6·0	275·9	86·3	1878·6

β Aquarii. Cycle, No. 786.

	A-B °	″	
*H_2.	322·8	20·0	IV. Cat.
Sm.	320·0	25·0	1833·73
Bu.	320·0	34·26	1877·7

20 *Pegasi. Cycle*, No. 799.

	A-B °	″	
*H_2.	320·0	40	H., I. Cat.
Sm.	330·0	35·0	1838·66
Bu.	326·1	51·3	1877·7

Alternative Measures of Distance from the MSS.

	Δ R.A. s		Δ R.A. s		Δ R.A. s
β Cassiopeiæ	19·1	α Orionis	6·8	α Canis Minoris	18·3
α Andromedæ	4·57	μ Geminor.	5·4	α Boötis	15·1
14 Arietis, A–C	3·8	β Can. Maj.	4·7	δ Boötis	7·9
o Ceti	7·8	ζ Gem., A–B	1·8	δ Aquilæ	6·6
δ Persei	7·0	,, A–C	5·0	α Cephei	9·8 (w_1)
257 P. IV. Tauri, A–C	4·3	δ Can. Maj.	13·5	,,	8·8 (w_2)
γ Orionis	3·6	30 Can. Maj.	5·6	4 Cassiop., A–C	25·3
β Leporis	13·1	η Can. Maj.	13·4	,, A–B	7·4
ε Orionis	9·8	63 Geminor.	1·8	γ Cephei, about	44
124 Tauri, A–B	5·5	145 P. VII. Argûs	1·5	171 P. XXIII. Androm., A–C	8·4
,, A–D	1·0				

Errata (from the List issued by Admiral Smyth).

"A rigorous re-examination of Captain Smyth's *Cycle of Celestial Objects* has led to the detection of the following typographical errors. These the Amateur-Astronomer is earnestly requested to correct in his copy with a pen, in order that they may occasion no loss of time to him.

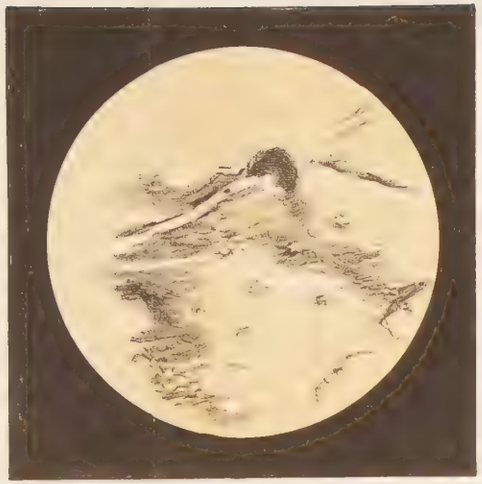
Volume II.

Page 25, No. xxxix., *for* R.A. $0^h\ 57^m\ 23^s$ *read* $0^h\ 57^m\ 39^s$.
,, 33, No. xlix., *for* Dec. 57° 56′·9, *read* 57° 28′·9.
,, 34, No. l., l. 4, *for* Epoch 1830·39, *read* 1830·89.
,, 46, No. lxxiv., *for* Dec. 37° 27′·9, *read* 36° 27′·9.
,, 54, No. lxxxvii., *for* R.A. $2^h\ 1^m\ 16^s$, *read* $2^h\ 1^m\ 12^s$.
,, 67, No. cix., *for* R.A. $2^h\ 33^m\ 8^s$, *read* $2^h\ 33^m\ 18^s$.
,, 94, No. cliii., *for* R.A. $3^h\ 59^m\ 6^s$, *read* $3^h\ 59^m\ 11^s$.
,, 115, No. clxxxviii., *for* Dist. of AC = 15″·1, *read* 15″·0.
,, 152, No. ccli., *for* R.A. $6^h\ 22^m\ 45^s$, *read* $6^h\ 22^m\ 25^s$.
,, 199, No. cccxxix., *for* R.A. $8^h\ 30^m\ 31^s$, *read* $8^h\ 30^m\ 41^s$.
,, 227, No. ccclxxiv., *for* Mean N.P.D. (l. 5) $77^h\ 15^m\ 12^s{\cdot}43$, *read* 77° 15′ 12″·43.
,, 256, No. ccccxxii., *for* Leo, l. i. of Description, *read* Ursa.
,, 289, No. ccccIxviii., *for* R.A. $12^h\ 51^m\ 30^s$, *read* $12^h\ 51^m\ 26^s$.
,, 300, No. ccccIxxxi., *for* μ Hydræ, *read* u.
,, 335, No. dxxxiii., *for* 22 M., *read* 102 M.
,, 371, No. dlxxxv., l. 2, *for* 15^h, *read* 16^h.
,, 392, No. dcxvii., *for* Σ's ∠ 76° 77′, l. 32, *read* 76°·77.
,, 411, No. dcxxxvii., *for* ∠ in 1842·39 = 255°·0, *read* 259°·5.
,, 442, No. dclxxxii., *for* Dec. 35° 15′·2, *read* 32° 15′·2.
,, 453, No. dccvi., *for* 51 Ḫ V. *read* 51 Ḫ IV.
,, 454, No. dccx., *for* 36 Ḫ V., l. 2, *ab imo*, *read* 46 Ḫ V.
,, 534, No. dcccxxxviii., *for* Dec. 64° 24′·3, *read* 61° 24′·3.'

I.

1878, Oct. 17, 12h. 20m., G.M.T.

III.



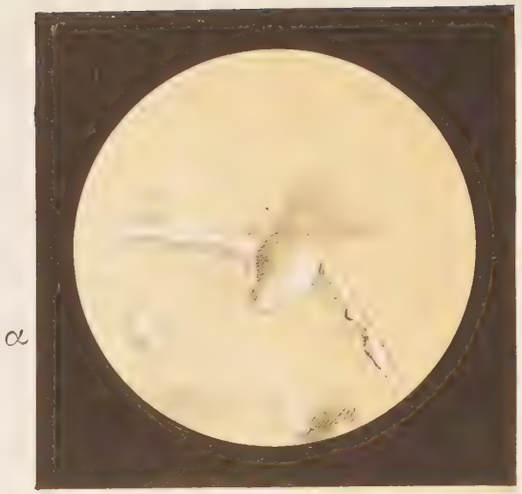
Oct. 17, finished 15h. 15m., G.M.T.

V.

Oct. 17, finished 16h. 20m., G.M.T.

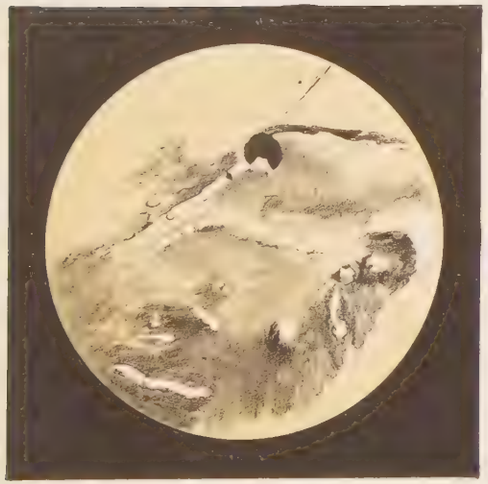
VII.

α



1878, Nov. 4, 7h., G.M.T., Def. good, power 307.

II.



1878, Oct. 17, 13h. 40m., G.M.T.

IV.

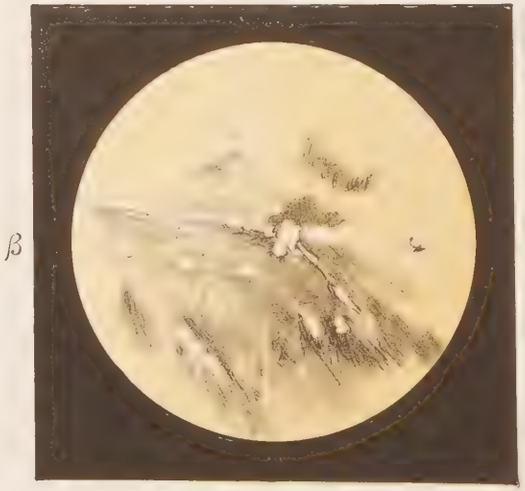
Oct. 17, 15h. 35m., G.M.T.

VI.

Oct. 17, finished 17h., G.M.T.

VIII.

β



1878, Nov. 4, 10h. 20m., powers 307, 442.

W.H. Wesley, Lith

Malby & Sons, Imp

1878, Dec. 10, 12h., G.M.T., powers 307 & 442.

W.H.Wesley, Lith.

Malby & Sons, Imp.

Postscript.

I have received the two following measures by letter from Mr. Burnham, too late for insertion in the body of my paper.

β Andromedæ. Cycle, No. 43.

	A–B °	″	
Sm.	299·0	225·0	1839·5
Bu.	293·6	297·9	1878·9 (single distance)

δ Persei. Cycle, No. 135.

	A–B °	″	
Sm.	315·0	140·0	1833·7
Bu.	313·3	108·62	1878·9

I do not know of any other measures. Mr. Burnham's are made with the magnificent Dearborn refractor of $18\frac{1}{2}$ inches aperture.

Clapham,
1878, *December* 18.

Note on some hitherto Unnoticed Features near the Lunar Crater Hyginus. By Lord Lindsay and Dr. R. Copeland.

It may be as well to preface the following note by the statement that it is not the intention of the writers to prove or disprove the existence of active volcanic agency in the Moon at the present moment. The sole object in view is to place on record certain not uninteresting observations confirming the well-known fact that the neighbourhood of the lunar crater *Hyginus* is full of complicated shallow irregularities and strongly marked differences of tone, which tend together to produce great apparent changes of surface configuration with change of illumination, and further to show that there exist striking features in the immediate neighbourhood which have hitherto escaped clear detection, but of which some traces may be found in the comparatively old map of Lohrmann.

During the night of October 17, 1878, the terminator passed over *Hyginus*. The whole night was fine, and the definition good, at times exceptionally so.

Under these circumstances the configurations of the region adjoining Schröter's well-known Rille were revealed in a very full degree. Six drawings were made between $12^h\ 20^m$ and 17^h G.M.T. Reserving for a future occasion a fuller discussion (based on micrometrical measures) of all these sketches, special attention is drawn to Nos. I, III, V, and VI.

Sketch No. I, which is a general representation of everything that is visible near *Hyginus*, exhibits no trace of a crater to the S.W. of *Hyginus*; but in No III, taken at $15^h\ 15^m$, a crescent-shaped ridge is shown abutting on that side of the crater.

At this time the observers were unable to interpret the meaning of this curved ridge; but by $16^h\ 15^m$, Sketch No. V,* it had become obvious enough that the ridge formed part of the wall of a shallow but very regular crater, which may be called No. 29 (*see* key map), immediately S.W. of *Hyginus*. In the same direction there was also a second, somewhat smaller, though most obvious crater.

These two craters seem never to have been previously observed. The south-westernmost of them must have disappeared in the shadow of night almost immediately after it was first seen; for a sketch made at 17^h (No. VI) shows no trace of it. The written observations also expressly confirm this fact.

On the other hand, its larger neighbour had become the most conspicuous object in the whole district: *Hyginus* being merged into the darkness of a large shallow depression to the N.E. that had been gradually making itself more and more obvious during the course of the night. On November 4, 1878, the region was examined under a moderately high illumination. It was then seen that the western end of a dark patch S. of *Hyginus* formed, and was coterminal towards the west with, the floor of No. 29; and, indeed, on November 8, one of us, in conjunction with Mr. G. J. Lohse, the Assistant Astronomer, saw most clearly that the western end of the above-mentioned shading was almost entirely, or perhaps completely, separated by a sort of strait from the main portion of the marking. This feature was again most clearly seen on December 10, at midnight. (Outline sketch).

This shaded patch seems to be visible under all except the very lowest illuminations, and is sufficiently well shown in Lohrmann, Sec. I., to convince us that it has, not greatly changed since his day. It is therefore highly improbable that Crater No. 29 has been formed since the date of Lohrmann's observations.† Yet the shading is not given by Schmidt or in the *Mappa Selenographica*. Possibly it may be shown in the large-scale map of this region by Mädler, which we have not had an opportunity of examining. It is given, however, in Dr Klein's Map, in *Sirius*, No. 8, 1878, but of a shape differing considerably from that in which we see it—Sketches of November 4 and December 10.

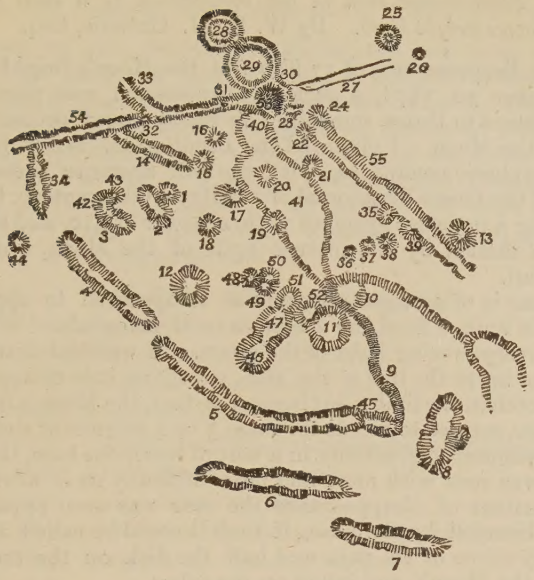
These facts show with what extreme caution all presumed evidence of change on the Moon's surface ought to be received, and how necessary it is to accumulate observations made under various, and particularly *low*, illuminations.

The observations were made with powers 307 and 442, on a refractor of 15·06 inches aperture.

Dun Echt Observatory,
1878, *Dec.* 11.

* Note to Sketch V. Finished about $16^h\ 20^m$ G.M.T., at which time the shallow crater No. 29 was as black as *Hyginus*, and the peak No. 20 had disappeared.

† The *Topographie der sichtbaren Mondoberflächen*, Abt. I., containing Sections I.-IV., was published in 1824.



Description.

Reference No. to Key-map.	No. of Sketch in which the object is shown.						Remarks.
1	I	II	III	IV	V	VI	Seen as a crater in 1, 2, and 3.
2	I	II					
3	I	II					
4	I	II					
5–8	I						
9	I	II			V	VI	A deep valley.
10	I	II			V	VI	
11	I	II			V	VI	
12	I	II			V	VI	
13	I	II					And probably in V and VI.
14			III				A very low ridge.
15			III	IV	V		A small hill.
16			III	IV			A very low hill.
17			III	IV	V		A hill.
18			III		V		A low hill.
19			III	IV			A rise in edge of depression.
20		II	III	IV	V		A hill in a depression.
21			III	IV	V	VI	A hill on table-land.
22					V	VI	Elevation on edge of Rille.
23					V	VI	„ „ „

11, 52, 51, 47, and 46 are portions of Mädler's *Hyginus β*.
25 is Mädler's *Hyginus b*.
The Rille, 27, was discovered by Lohrmann.

Note on a Phenomenon seen in the occultation of a Star at the Moon's bright limb. By W. H. M. Christie, Esq.

At the disappearance of 17 *Tauri* at the Moon's bright limb, on November 10, 1878, a curious phenomenon was presented, which appears to throw some light on the apparent projection of a star on the Moon. I observed the occultation, under favourable atmospheric conditions, with the full aperture (nearly 13 inches) of the Great Equatoreal of the Royal Observatory, Greenwich, using a negative eyepiece with a power of 310, and my eye was much dazzled by the intense light of the Moon, only 19^h past the full.

The star is of 4 magnitude, and as it appeared to approach the Moon's limb, I fixed my attention on it, being afraid of losing it in the overpowering light of the Moon. I watched it steadily till it came up to the line of the limb, expecting it to disappear at a slight notch in the limb; but instead of that, the Moon's limb, to my surprise, seemed to recede for some 3 or 4 seconds of time, and the star disappeared gradually in a sort of luminous haze, through which it was seen with more and more difficulty as it advanced. At the instant of disappearance the star was seen apparently perfectly bisected by the limb, if limb it could be called, that is, completely shorn of its rays and half the disk on the one side (towards the Moon), and intact on the other.

This observation appears to accord perfectly with the explanation of the phenomenon of projection which the Astronomer Royal has given in vol. xxviii. of the *Memoirs*. The star was not bright enough to be seen distinctly projected on the Moon's disk, but was yet not so faint as to be overpowered by the irradiation at the limb. A spurious limb being formed, as the Astronomer Royal has explained, by the superposition of diffraction images, the intensity of light would increase gradually from the spurious to the true limb. At the former the star's light overpowered that of the Moon; at the latter the Moon's light overpowered that of the star. The circumstance that the diffraction image of the star (disk and rays) was cut off completely on the side of the Moon at the instant of disappearance is in perfect accordance with the Astronomer Royal's explanation.

At the disappearance of 20 *Tauri*, a star of 5 magnitude, I tried the effect of a graduated dark wedge (neutral tint) to diminish the glare of the Moon, and was enabled to observe the occultation perfectly. The star disappeared when in apparent contact with the bright limb, but not instantaneously, as in occultations at the dark limb. At the occultation of 16 *Tauri*, a star of 6·7 magnitude, the star was lost in the glare of the Moon before coming up to the limb.

Royal Observatory, Greenwich,
1878, *December* 9.

On a New Variable Star, ι Andromedæ. By J. E. Gore, Esq.

From my own observations during the last three and a half years I am confident that this star is variable to the extent of about half a magnitude. It is sometimes distinctly *brighter* than κ *Andromedæ*, and sometimes quite as distinctly *fainter*. The star has been estimated 4 mag., by most of the authorities, but it was rated as low as 7 mag. by Bradley and Piazzi, and it is 6 mag. in Harding's *Atlas*. D'Agelet rated it once 3–4 mag., and once 7 mag. Heis gives ι 4 mag. and κ 4–5 mag. The following are my observations :—

May 1875.	ι very slightly fainter than κ *Andromedæ*.
Aug 1875.	Same relative brilliancy.
Jan. 30, 1876.	Very slightly brighter than κ.
Oct. 1876.	A little less than κ (about $\frac{1}{4}$ mag.)
Dec. 13, 1876.	ι slightly brighter than κ.
Feb. 10, 1877.	ι slightly but distinctly brighter than κ *Andromedæ*.
Feb. 16, 1877.	ι and κ almost exactly equal.
Aug. 4, 1877.	ι very slightly less than κ.
Aug. 23, 1877.	ι almost exactly equal to κ—during totality of lunar eclipse.
Aug. 31, 1877.	ι slightly, but decidedly, less than κ.
Oct. 30, 1878.	ι only just perceptibly less than κ.

I have not been able to determine the period, but it would seem to be tolerably short.

Dromard, Co. Sligo,
November 19, 1878.

On a Variable Diaphragm for use in Solar and Sidereal Observations. By F. W. Levander, Esq.

The instrument which I have the honour to bring to the notice of the Society had its origin in a desire on my part to devise a plan which should possess most, if not all, of the advantages of a Dawes' eye-piece, but capable of being produced at a smaller cost. It consists of an adapter to slide into the tube, as an ordinary eye-piece, and of a draw-tube to receive any eye-piece; between these there is a diaphragm, which, by one motion of the milled screw, is capable of being opened to the greatest extent required, or of being entirely closed. For solar work a draw-tube slides in such a manner that the interchangeable dark glasses with which it is furnished are at some distance within the focus of the object-glass. To prevent time being lost during an observation by interchanging these, it was proposed to adopt a revolving disk of dark glasses, as in Dawes' instrument; but the use, which has been kindly suggested, of a neutral tint wedge is,

I think, the best plan. By these means the light can be diminished to any extent, and with the diaphragm any part of the solar disk can be isolated and examined.

The instrument will also be found to be of great use in examining or detecting faint points of light when near brighter objects, as double stars of very diverse magnitudes, faint satellites of planets, lunar formations, &c., by reducing the glare of the surroundings and obviating the inconvenience of using the edge of the object-glass or speculum or the introduction of a field-bar. It also serves for measuring the vanishing point of stars, for testing magnitudes.

The screw-head is divided into equal parts, and, by noticing the reading when a particular observation was made, it may be recovered—under similar atmospheric conditions—at a future time. By a simple arrangement, the diaphragm may be placed out of the centre, so that, by revolving the instrument, the space round an object in the centre of the field of view may be examined.

I have to thank Messrs. Horne & Thornthwaite for the care with which they have carried out my design.

7 *Chalcot Crescent, Regent's Park, N.W.,*
December 11, 1878.

Explanation by Captain Bigg-Wither in regard to his Observation of the Transit of Venus, see *Monthly Notices,* Vol. xxxviii., foot-note, p. 433.

The longitude ($4^h\ 48^m\ 1^s{\cdot}8$ E.) given for Mooltan was not taken from the survey, but was the most probable mean of the results of 15 occultations of stars by the Moon; so that the correction of $+2^s{\cdot}3$ made by Captain Tupman is inadmissible. The local time for all these observations, as well as for the Transit of *Venus*, was obtained by a 24-inch transit instrument and a sidereal chronometer, and was in every case known certainly within half a second of the truth. The occultations were observed with the same telescope as that used for the Transit of *Venus*, and were all taken from the "Elements" in the *Nautical Almanac*; they were reduced, using the Moon's tabular places as given in the *Nautical Almanac*.

Mooltan,
December 9, 1878.

Errata.

P. 37, line α, col. E of Table II., *insert* 581.

„ „ η, „ 3 „ III., *for* ·1 *read* ·15.